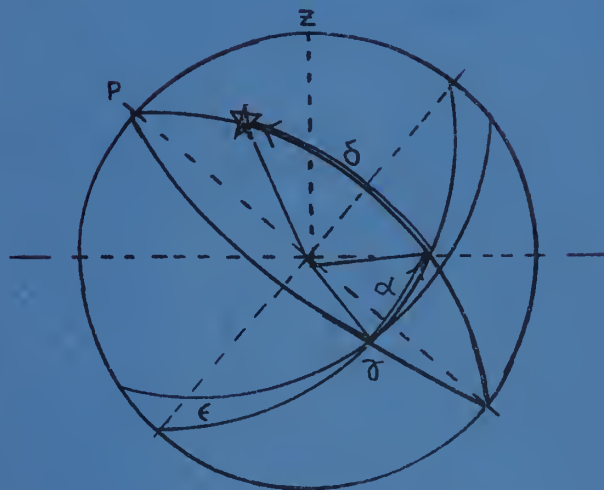


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DETERMINATION
OF THE
ASTRO-GEODETTIC DEFLECTION
OF THE
VERTICAL



LT. J. E. BENNETT, USN

1962

Thesis
B375

THE DETERMINATION OF THE ASTRO-GEODETIC
DEFLECTION OF THE VERTICAL

A Thesis

Presented in Partial Fulfillment of the Requirements
for the Degree Master of Science

by

Joseph Evans Bennett, B.S.

//

The Ohio State University
1962

Approved by

Adviser
Department of Geodetic Science

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1. INTRODUCTION

Many centuries past man began to speculate about the shape of the earth when he first became aware that it was not a flat surface with terrain superimposed upon it.

About 582 BC the first good notion of the shape of things came from Pythagorus, who expounded the idea of a spherical earth and universe - an idea concurred in by Aristotle some two-hundred years later. The key to a reasonable idea of the shape of the earth was undoubtedly the realization of a certain orderly relative motion between the earth and the celestial bodies. Thus we can say that a knowledge of astronomy has been of prime importance to geodetic science since its inception.

About 200 BC, Eratosthenes brought these ideas under scientific investigation by attempting to measure the size of the earth. Inasmuch as Geodesy is defined as the branch of applied mathematics concerned with measuring or determining the shape of the earth, Eratosthenes is known as "The Father of Geodesy." Although his measurements were crudely made, using the average speed of a camel caravan over a period of days to measure distance over the ground, and the angle of the sun at noon to measure the astronomic latitude, his determination of the length of a meridian circle was only 15% too large.

From the time of Eratosthenes to that of Newton, some 1800 years later, Geodesy was primarily concerned with more accurate measurement of the earth on the assumption that it was a sphere. Newton's studies of the earth and the universe led to an indication that the earth's general shape was that of an ellipsoid.

Between 1735 and 1741, the French Academy of Science sent parties to Peru and Lapland to measure a 1° arc on the ground, with the results indicating that the more northerly arc was of greater length; on the basis of these investigations, the earth was determined to be an ellipsoid of revolution about its minor axis, with a value of flattening of 1:310.3.

At the Madrid convention of the I.U.G.G. in 1924, world geodesists and geophysicists adopted the ellipsoid of revolution determined by Hayford in 1910 as a standard. This ellipsoid is defined as having a semi-major axis of 6,378,388 meters and a value of flattening of 1:297.0; although this remains the internationally accepted standard, more recent determinations indicate that a value of flattening of 1:298.3 is better. Still more recent investigations have led some geodesists to present the theory that the earth is a triaxial ellipsoid. The third axis being in the equator plane about 200 meters longer or shorter than the semi-major axis, which would correspond to a flattening of about 1:31891.94. If we picture the earth as a ball twenty-five



feet in radius, the international ellipsoid would have a polar radius (semi-minor axis) of 24.916 feet. Now if we determine the third axis, the corresponding value on our ball would be a radius of 24.9992 feet. For practical purposes, therefore, we consider an ellipsoid of revolution to be the closest mathematical expression for the shape of the earth.

While the adoption of a standard ellipsoid (also called an oblate spheroid, or more simply a spheroid) is of tremendous value, there is no simple, direct method of reducing surface surveys to this reference surface. To overcome this difficulty, surface surveys can be reduced to mean sea level by including level observations to determine departures from mean sea level.

The surface defined as mean sea level is termed the geoid, which departs from the spheroid as much as several hundred feet in elevation and up to one minute (arc) in inclination. In order to reduce our observations from mean sea level to the reference spheroid, therefore, we must be able to determine the deviation between the geoid and the spheroid in both elevation (undulation of the geoid) and inclination (deflection of the vertical).

The general procedure here is to establish a geodetic datum based on an arbitrary separation (N) between the spheroid and the geoid at an origin point. (Refer to Figure 1)

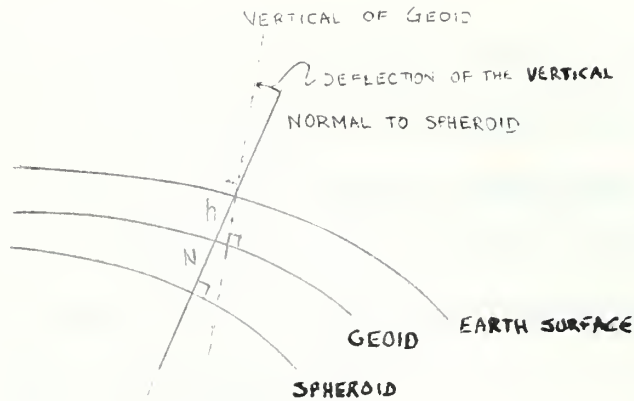


FIGURE 1.

Then the elevation (h) of that point above mean sea level (obtained by leveling) is readily converted to a height above the spheroid, and the geodetic latitude and longitude of this origin point is assumed to be correct. Astronomical latitude and longitude are next determined by star observations and the deflection of the vertical (from the normal to the spheroid) is computed. By thus selecting the height of the origin point above the spheroid and the deflection of the vertical, the center of the spheroid is defined.

Running survey nets from this origin point we may now readily convert ground coordinates to geodetic coordinates if we assume that the separation of the geoid and spheroid is everywhere the same. In actuality the separation is not everywhere the same, but any change in this separation may be deduced from a change in deflection of the vertical. It is necessary, therefore, to make periodic determinations of this deflection of the vertical as we proceed from our origin point in order to define the shape of the geoid.

Such a datum has been established in the United States

at Meade's Ranch, and in Europe at Potsdam, and the geodetic coordinates of points in North America and Europe are thus defined, respectively, on non-coincidental spheroids. This presents difficulties in correlating the relative positions of inter-continental points in the absence of intercontinental geodetic ties. (Refer to Figure 2)

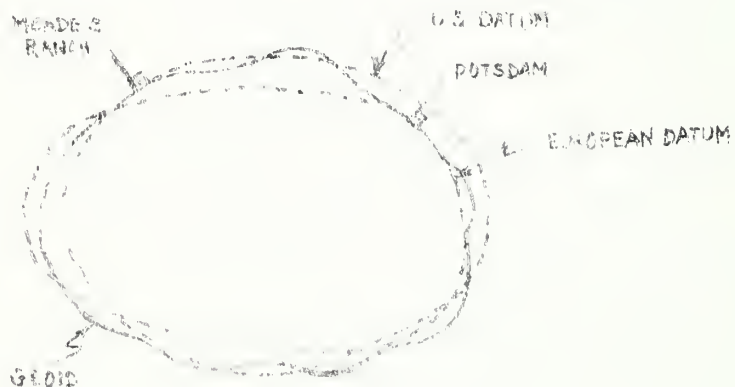


FIGURE 2.

With the rapid advance of technology making available long range electronic distance measuring devices, artificial earth satellites, and high speed data processing machines, together with a public awareness of the necessity for more precise knowledge of the shape of the earth and commensurate financial support, a solution to this problem may be forthcoming in the near future.

The scope of this thesis, however, is limited to a discussion of the determination of deflection of the vertical by comparison of geodetic and astronomic coordinates at a

point, and is best understood by discussion of the specific problem in the following chapters.

2. ASTRONOMIC DETERMINATIONS

2.1 - General

For practical purposes, we assume that all the stars in the sky are situated on a sphere of infinite radius whose center coincides with the center of the earth. The position of stars on this sphere is defined by a coordinate system whereby the north celestial pole lies on the rotation axis (extended) of the earth, directly above the earth's north pole and in the vicinity of the North Star (Polaris); the celestial equator is defined by the intersection of a plane through the earth's equator with the celestial sphere. Celestial parallels are then defined by angular displacement along great circles from their intersection with the celestial equator (0°) Northward (+) and Southward (-) toward the celestial poles ($\pm 90^\circ$): this coordinate is termed "DECLINATION" and is symbolized by δ .

The second coordinate required to fix the position of a star on the celestial sphere corresponds to longitude on the earth and defines the angular displacement (Eastward) of the great circle through the star and the poles from a selected great circle (0°) through the poles and the First Point of Aries (γ). γ is defined as the position occupied by the sun at the vernal equinox, and represents the intersection point of the ecliptic plane with the equator plane

on the celestial sphere (occurring about March 21 each year).
(Refer to Figure 3)

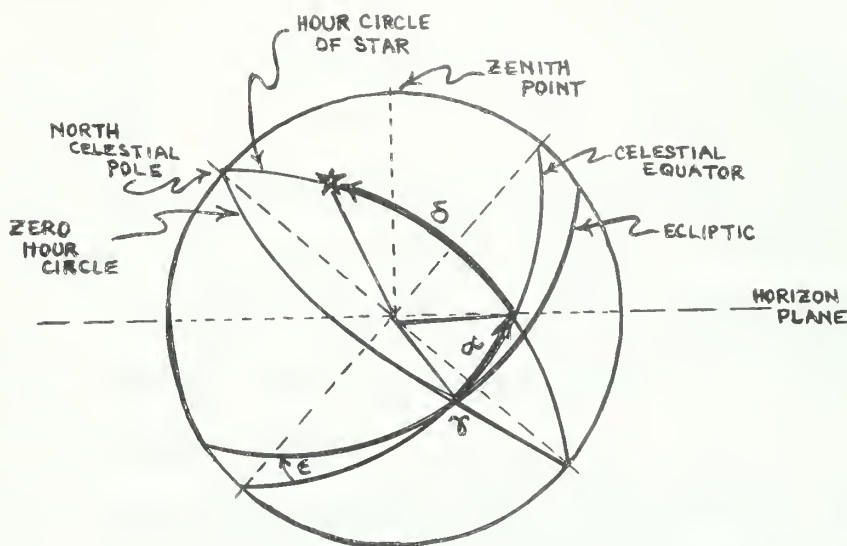


FIGURE 3.

The second coordinate, termed "RIGHT ASCENSION", is symbolized by RA or α ; RA is usually measured in hours rather than in degrees, zero hour (sidereal time) occurring at any point on earth at the instant the zenith point intersects the hour circle of γ . Since the RA of any star represents its number of hours departure from γ , the local sidereal time at any point thus may be determined by noting the instant the hour circle of a star intersects the zenith point; more simply stated, by noting the exact instant the star transits the meridian of the observing point. By referring to a catalogue for the RA of the observed star, then, we have the local sidereal time in hours; if we also know the Greenwich sidereal time at the instant we observed the star's transit, then we may readily compute our astronomic longitude



Fig. 1.

The diagram shows the distribution of the population of the region in 1950. The population is concentrated in the central part of the region, with a density of 100 persons per square kilometer. The population density decreases towards the periphery of the region. The diagram also shows the distribution of the population by age and sex. The population is divided into three age groups: 0-14, 15-64, and 65 and over. The population is also divided into males and females. The diagram shows that the population is relatively young, with a high proportion of the population in the 0-14 age group. The population is also relatively balanced in terms of sex, with a slight excess of males.

with respect to Greenwich.

Similarly, if we know the exact declination of a star and it transits precisely at our zenith point, we may say that our astronomic latitude is precisely equal to the declination of that star. In practice, a star rarely transits precisely on our zenith point but if we can measure the zenith distance (Z.D.) of the star at the instant of transit, we may determine our astronomic latitude as being equal to the declination plus (minus) the Z.D. of the star to the North (South) of our zenith point.

All the preceding assumes that \odot and all the stars are fixed on the celestial sphere, and that the earth rotates at a constant speed about its axis. In actuality, however, \odot does not always occur at the same point due to a slight variation in the obliquity of the ecliptic (ϵ). All the stars are not at the same distance (from the center of the celestial sphere) and consequently will have variations in their coordinates of Right Ascension and Declination from this source as well as apparent changes due to the non-constant rotation of the earth about its axis, "wobble" of the rotation axis, and refraction of light as it passes through the earth's atmosphere. The effect of "wobble" in the earth's axis is an apparent change in the position of the celestial poles and the celestial equator, and the joint effect of this change in the celestial equator combined with the variation in obliquity of the ecliptic is a change in the position of the vernal equinox, and consequently, apparent



changes in the coordinates of all the stars. This motion of γ is toward the west and is called Precession of the Equinox, which is broken into two parts:

1. Precession: uniform westerly motion of about $50''$ 2 per year
2. Nutation: small periodic variation of the motion of the equinox

The changes in the right ascensions and declinations of stars are generally divided into groups of very slow changes (such as Precession) called Secular changes, and those which occur more rapidly (such as Nutation) called Periodic changes.

The apparent inter-stellar motion due to their varying distances is a secular change termed Proper Motion. Another apparent change in the observed position of a star is termed Aberration and is somewhat analagous to the "Lead" angle applied when firing a gun at a moving target; Annual Aberration is due to revolution of the earth in its orbit, while Diurnal Aberration is due to rotation of the earth about its axis. Annual Parallax is an apparent change in the position of a star due to the annual revolution of the earth (and therefore of the center of the celestial sphere).

The Apparent Place of a star, then, is the position it is actually sighted in with respect to the center of the earth. The Mean Place of a star is obtained by applying corrections for precession, nutation, aberration, proper motion and annual parallax to the Apparent Place, and the

converse also is true. For our purposes, the apparent place of a star is used in the determination of our Astronomic latitude and longitude.

2.2 - The Observations

The data used in this portion of the thesis were extracted from observations made in the field course in Geodetic Astronomy (687) during the summer of 1961. The directions for making the star observations are detailed in the USC & GS Special Publication No. 237 ([8], pp. 36-38 for longitude and pp. 64, 66-72 for latitude), with additional details for the instrument taken from Odermatt [10]. The observation technique is touched upon but briefly here, the emphasis being on the computations and their results.

The observation point "Astro Pillar, OSU Farms" is a poured, reenforced concrete column about 16 inches square and 10 feet long, with about 4 feet extending above ground level. A brass marker is countersunk into the top center and center-punched for the plumb reference of these observations. The pillar is situated about 1000 feet south of Lane Avenue and about 1900 feet east of North Star Road. It is most easily reached via a gravel road which intersects Lane Avenue about 0.9 miles west of the Lane Avenue-Olentangy River Road intersection. The southeast corner of this gravel road intersection is occupied by a green-shingled farm house, while the southwest corner is occupied by a small fenced-in area con-

taining weather measuring instruments. There are three smaller concrete monuments along the west side of this road which define a base line for university use, but the Astro Pillar is considerably larger than these, and about 8 feet west of the base line. Figure 4 is a sketch of the described location.

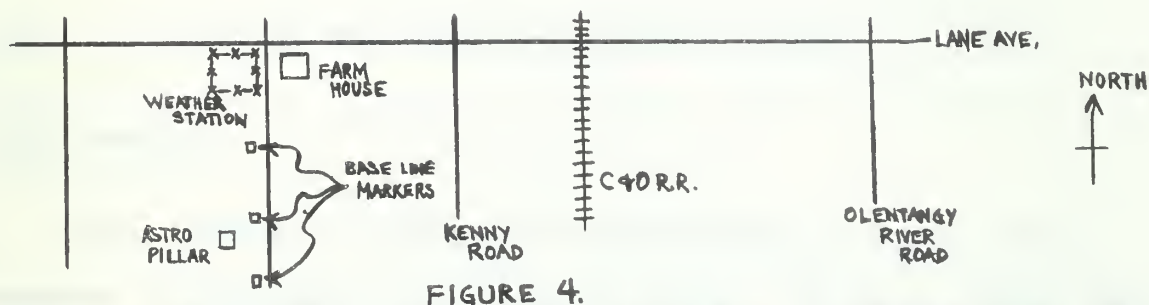


FIGURE 4.

The instrument used for all astronomic observations in these calculations was the Wild T-4, and the instrument constants were determined to be as follows:

$$R = \text{One Turn of Micrometer} = 10^S.1849 \pm 0^S.0112$$

$$m = \zeta = \text{Micrometer Lost Motion} = 0^P.031$$

$$s = \beta = \text{Width of Contact Strips} = 1^P.114$$

$$c = \frac{1}{2} (\zeta + \beta) = 0^P.572$$

$$\text{Where: } 1^P = 1/100 \times R$$

The method of obtaining these constants is described in detail in [8], pp. 24-27, using the symbols m and s in lieu of ζ and β , as indicated above. The level values were obtained according to the procedure described in [8], pp. 23 and 24, from level trier readings by Dr. I. I. Mueller and Mr. L. Sukman at the USC & GS labs in Washington, D.C. and were found to be:

<u>LEVEL NO.</u>	<u>DESCRIPTION</u>	<u>VALUE OF ONE DIVISION</u>
616	Hanging Level	0".89716 \pm 0".003
2182	Horrebow Level	0".918 \pm 0".049
2616	Horrebow Level	1".045 \pm 0".036

The application of these constants will be pointed out in the appropriate places.

2.3 - Latitude By The Horrebow-Talcott Method

2.31 General

This method of latitude determination is widely used because it is simple, fast and accurate. In this method the star program (Table I) is set up so that a pair of stars may be observed to transit, one north and one south of the zenith, without changing the inclination of the instrument. By so doing, we are able to measure the difference in the zenith distance of the two stars by the impersonal micrometer, thus avoiding personal errors in sighting. Additionally, the requirement of approximately equal zenith distances imposes the advantage of having approximately equal refraction of the ray path of each star. Finally, the precision (Horrebow) levels will indicate any variations in the meridional inclination of the instrument, and application of the constants (see 2.2) will enable us to make accurate corrections to our observations. The words are best put in [8] p.64:

The high degree of accuracy attained by the Horrebaw-Talcott method follows from the fact that the method depends entirely on differential measurements.

The criteria for selecting stars to satisfy such a program are listed in detail in [8] pp. 64, 67-68, and are dependant upon the field of view of the instrument (about 20" for the T-4), the time required to observe each pair (about 4 minutes), and the time interval during which observation conditions may be considered constant (limit 20 minutes difference in R.A. for stars of a pair).

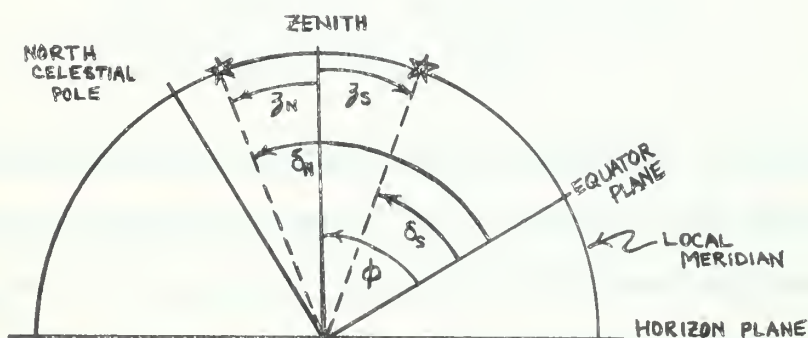


FIGURE 5.

Referring to Figure 5, it is easily seen that:

$$\phi = \delta_N - z_N$$

$$\phi = \delta_S + z_S$$

$$\text{therefore: } 2\phi = \delta_N + \delta_S - z_N + z_S$$

$$\text{and if: } z_N = z_S, \text{ then: } \phi = \frac{1}{2}(\delta_N + \delta_S)$$

In practice, the zenith distance of the north star (z_N) is rarely equal to that of the south star (z_S). The instrument is sighted to the mean zenith distance ($z_m = \frac{z_N + z_S}{2}$) and corrected by the difference ($\Delta z = z_N \pm z_m = z_S \mp z_m$).

Referring to figure 6:

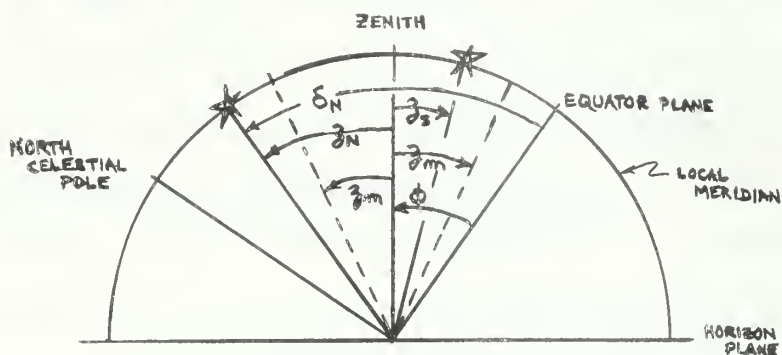


FIGURE 6.

$$\phi = \delta_N - (z_m + \Delta z)$$

$$\phi = \delta_S + (z_m - \Delta z)$$

$$2\phi = \delta_N + \delta_S - z_m + z_m - 2\Delta z$$

$$\phi = \frac{1}{2}(\delta_N + \delta_S) - \Delta z$$

In our observations we record the micrometer reading for the star with the eyepiece west (M_W) and for the other star with eyepiece east (M_E) and find the difference between them ($M_W - M_E$) is $2\Delta z$, in units of micrometer turns. The correction applied is therefore $= \frac{1}{2}R(M_W - M_E)$, where R = the value of one turn of the micrometer in seconds of arc.

A further correction must be made because of the unequal refraction of light due to the difference in the zenith distances. This correction is also a differential one, and is of the same sign as the micrometer difference. If the refraction at z_N is equal to r , and the refraction at z_S is equal to r' , it is apparent that the correction to be applied is: $\frac{1}{2}(r - r')$. The proof is similar to that for the micrometer correction; where the apparent z is greater than

z_m the effect of refraction is to decrease Δz , and where the apparent z is less than z_m the effect is to increase Δz . Reference [8], p. 176, Table VIII gives the value of this correction directly entering with arguments of z_m and one half the difference of the zenith distances, Δz .

The purpose of the Horrebow levels is to detect any small change in the selected z_m between the stars of a pair due to an inclination of the vertical axis. To find the center point of a single level bubble, the readings at each end are added and the sum divided by two; the same method is used to determine the center point with the level ends reversed. If the center point is not the same in both positions, the difference is divided by two to obtain the value of the correction necessary to re-level the platform. Alternatively, the sum of the readings in the second position may be subtracted from the sum of the readings in the first position and the difference divided by four to obtain the necessary correction. Since there are two Horrebow levels, the sum of the four readings with the eyepiece east ($\sum L_E$) is subtracted from the sum of the four readings with the eyepiece west ($\sum L_W$), and the difference then divided by eight to obtain the desired correction in terms of level divisions. In practice, no adjustment is made, but the correction is converted to seconds of arc by multiplying by the mean value of one level division, and the result applied to the observed declinations. This correction is symbolized:



$$\left(\frac{d + d'}{2} \right) \left(\frac{\sum L_W - \sum L_E}{8} \right)$$

where: d = value of one division of one Horrebow level

d' = value of one division of the other Horrebow level

$\sum L_W$ = sum of the four level readings with eyepiece west

$\sum L_E$ = sum of the four level readings with eyepiece east

(the values of d and d' are given in section 2.2)

Finally, if a star is not observed on the meridian it will not be at its maximum altitude and the apparent declination will be too small. Reference [8], p. 177, Table X gives the value of the correction directly, entering with arguments of declination and time error of the right ascension. If the star is within 6^S ($90''$) of the apparent right ascension, the maximum correction for the star is 0^m01 and may be neglected. If the meridian corrections for the pair of stars are designated m and m' , the correction to the observation is symbolized: $\frac{1}{2}(m + m')$.

The complete formula for latitude using the Wild T-4 with this method is :

$$\begin{aligned} \phi = \frac{1}{2} (\delta + \delta') + \frac{1}{2} R(M_W - M_E) + \frac{1}{8} \left(\frac{d + d'}{2} \right) (\sum L_W - \sum L_E) + \\ + \frac{1}{2} (r - r') + \frac{1}{2} (m + m') \end{aligned}$$

2.32 Computation Of Mean Places

The Boss General Catalogue [4] lists the mean places for stars for the beginning of the year 1960.0 along with the annual and secular variations and the third term. The

first step is to reduce this mean place to the nearest beginning of a year. Since observations were made nearer the beginning of 1962 than 1961, we compute the mean place for 1962.0 according to the formulae from [8] p. 74:

$$\begin{aligned} R A (1962.0) = R A (1960.0) + \\ + (1962-1960) \text{ Annual Variation} + \\ + \frac{1}{2} (1962 - 1960)^2 (1/100) \text{ Secular} \\ \text{Variation} + \\ + \left(\frac{1962 - 1960}{100} \right)^3 \quad (\text{Third Term}) \end{aligned}$$

$$\begin{aligned} \text{Declination } (1962.0) = \text{Declination } (1960.0) + \\ + (1962 - 1960) \text{ Annual Variation} + \\ + \frac{1}{2} (1962-1960)^2 (1/100) \text{ Secular} \\ \text{Variation} + \\ + \left(\frac{1962 - 1960}{100} \right)^3 (\text{Third Term}) \end{aligned}$$

This computation is listed for each star in Tables II and III. The coefficient of the last term is 8×10^{-6} and this term may be neglected.

2.33 Besselian Star Numbers

The Besselian Star Numbers are dependent upon the mean right ascension (α_0), and mean declination (δ_0), and are defined in [1], p. 498:

$$a' = \cos \alpha_0$$

$$b' = -\sin \alpha_0$$

$$c' = \tan \epsilon \cos \delta_0 - \sin \alpha_0 \sin \delta_0$$

$$d' = \cos \alpha_0 \sin \delta_0$$

where: $\tan \epsilon$ = the value of the tangent of the mean

obliquity of the ecliptic, as listed in [1], p. 50.

The quantity μ' is the annual proper motion in declination as listed in the Boss Catalogue [4]. These constants are computed in Table IV.

2.34 Besselian Days and Day Numbers

The Besselian Day numbers are interpolated from tables in the Nautical Almanac ([1], p. 266) for the mean epochs of the observations. It is not essential to interpolate for each star, but limits must be imposed to obtain the required accuracy. For this purpose, I determined the rate of change per minute for each factor (A, B, C, D and γ) and found factors C and D to change most rapidly. Dividing 0".005 (desired accuracy) by the maximum c' and d' (obtained in Table IV) thus indicated the maximum allowable change for C and D, respectively. Dividing this limit by the rate of change per minute gave the maximum allowable time interval between transit time and mean epoch. This limit was determined to be 32 minutes based on values of C and c' , and 47 minutes based on values of D and d' . On this basis I established a limit of 25 minutes and grouped all the stars such that the mean value of their right ascensions would not deviate more than this from the R.A. of any star in a group. The mean epoch thus obtained (Sidereal Time) was reduced to Greenwich Sidereal Time using an assumed longitude of $5^h 32^m 10^s$ in each case. The Sidereal Times are converted

to Universal Times and the interval between them expressed as a decimal part of a day (see [1] pp. 464-465, and [8] p. 129). This decimal part of a day added to the Greenwich day gives the Besselian Day which is used to interpolate the Besselian Day Numbers for each mean epoch. The numbers A and B correct for precession and nutation, while C and D correct for aberration. The factor T is the decimal part of the tropical year between the epoch of observation and the nearest beginning year (1962 in this example).

2.35 Reduction to Apparent Declination

The reduction to Apparent Declination is shown in Table VI, and follows the method described in [1] p. 500 and in [8] p. 128. The formula is:

$$\delta = \delta_0 + T\mu' + Aa' + Bb' + Cc' + Dd'$$

where: δ = apparent declination

δ_0 = mean declination

T = decimal part of year from 1962.0

μ' = annual proper motion in declination

A,B,C,D = Besselian Day Numbers

a',b',c',d' = Besselian Star Numbers

In the various star catalogues there exist systematic differences in the tabulated mean places of a given star which will cause deviation in the computed apparent declinations (see Nassau [9] pp. 166-168). The FK3 catalogue is accepted as a standard and these apparent declinations should be corrected

from the Boss [4] system to the FK3 system by two more terms:

$\Delta \delta$ = change in declination due to declination

$\Delta \alpha$ = change in declination due to right
ascension

These corrections were not available at the time the apparent declinations were computed, but since they affect only the first term in this formula the mean value for each star pair may be added as a correction to the latitude of each pair. Similarly, the mean of these corrections to each pair may be added to the mean value of the latitude to obtain the same result that would be obtained using the FK3 system for the reduction to apparent declination. (See section 2.39).

2.36 Observation Data

The Observation Data are listed in Table VII. Column 2 indicates whether the star is North or South of the Zenith and column 3 whether the eyepiece is positioned East or West. Column 4 indicates the micrometer reading (in turns and divisions) representing the position of the star in the field of view at time of transit, while column 5 is the difference of these readings for each pair of stars. In symbols, column 5 is the quantity $(M_W - M_E)$, where M_W is the micrometer reading with eyepiece West and M_E is the micrometer reading with the eyepiece in the East position. Columns 6 and 7 are the readings of the North and South ends of the two Horrebow levels for each eyepiece position; there are 2 North and 2 South readings for each position East and West. Column 8 is

the difference: (sum of the 4 level readings with the eyepiece West) minus (the sum of the 4 level readings with the eyepiece East). In symbols, column 8 is $(\sum L_W - \sum L_E)$, in units of one level division. Column 9 is the chronometer time of observation.

2.37 Latitude Computation

The Latitude Computation in Table VIII is done by the following formula:

$$\begin{aligned} \text{Latitude} = & \frac{1}{2} (\delta + \delta') + \frac{1}{2} R (M_W - M_E) \\ & + \frac{1}{8} \left(\frac{d + d'}{2} \right) (\sum L_W - \sum L_E) + \frac{1}{2} (r - r') \\ & + \frac{1}{2} (m + m') \end{aligned}$$

Where: $\delta + \delta'$ = sum of the apparent declinations for a pair of stars in a set

R = value of one turn of the micrometer in seconds of arc = $15(10^s .1894) = 152'' .7735$

$(M_W - M_E)$ = west micrometer reading minus east micrometer reading (taken from table VII, column 5)

$(d + d')$ = the sum of the Horrebow level values

$(\sum L_W - \sum L_E)$ = sum of west level readings minus sum of east level readings (taken from Table VII, Column 8)

$\frac{1}{2} (r - r')$ = differential refraction taken from refer. [8] p.176 (Table VIII) with the same sign as the micrometer difference

m and m' are meridian corrections taken from reference [8] p. 177 (Table X). This correction is negligible when stars are observed within 6^s of the meridian.

It is important to note that the formula in reference [8] p. 72 is for the Bamberg broken-telescope transit and reverses

the sense of the corrections in that west readings (suffix W) are subtracted from east readings (suffix E). For use with the T-4, these readings must be taken as west minus east ($W - E$).

2.38 Chronometer Checks

Table X lists the comparison of the sidereal chronometer with the time signals from WWV on five megacycle frequency using the assumed longitude of $5^h 32^m 10^s$. The chronometer corrections and rates are used to correct observed chronometer time of transit. The chronograph was not used for radio signal to chronometer comparisons in this observation.

2.39 Summary of Latitude Computations

The latitude obtained for each pair of stars in Table VIII is adjusted in Table IX by the method of least squares as detailed in reference [8] pp. 78-81. An initial mean is obtained and any pair (set) having a residual of 3" or more is rejected. Because of uncertain level values in the case of stars 26986, 28956 and 30322 the sets containing them were rejected. In the adjustment we determine the deviation of the mean latitude from the most probable latitude: $c = 0''.010$. We also obtain a correction to our preliminary value for one-half turn of the micrometer ($\frac{1}{2} R$): $r = -0''.053$. The column headed Mr is the product of M and r for each set and is applied algebraically to latitude obtained in Table VIII and the result listed in column 6 of Table IX. A new mean

latitude is obtained, and represents the mean observed astronomical latitude of the station based on the observations in the Boss G.C.[4] system, without reduction to sea level.

The Apparent Place of a star will not be the same when derived from Boss [4] as when derived from the FK-3 system, and since the FK-3 System is accepted as a standard, the Apparent Place obtained from Boss [4] must be corrected by the quantities:

$\Delta \delta$ = change in declination due to declination

$\Delta \delta_{\alpha}$ = change in declination due to R.A.

I was unable to locate any definition of these quantities in the library and so consulted Professor Bobrovnikoff at McMillin Observatory. From our conversation I was able to learn only that differences in R.A. and declination for a given star do occur in different systems, primarily due to dissimilar observing conditions. In any event, the tabulated values for the corrections were obtained from my adviser, Dr. Mueller, and the corrections are applied in Table XI as a total correction to each set. The mean of the corrections is then applied to the mean latitude of Table IX. The final astronomical latitude, corrected to sea level, is $40^{\circ} 00' 13''.401$ North.

TABLE I

Station OSU FARM	Date 7 AUGUST 1961	Instrument T-4
ϕ : 40° 00'	2 ϕ : 86° 00'	Value of one turn: a: 0.387

[illegible]

* $(2\phi - \sum \delta)$ should not exceed 20 for any pair. The algebraic sum of $(2\phi - \sum \delta)$ should not exceed the number of pairs.



TABLE I continued

U. S. DEPARTMENT OF COMMERCE
COAST AND GEODETIC SURVEY

OBSERVING LIST FOR LATITUDE

Station OSU FARM Date 7 August 1961 Instrument T-4

ϕ : 40° 00' 2ϕ : 80° 00' Value of one turn: a: 0.387

Set
No.

9^x

10

11a

11b

12

13

14a

14b

Star No. Boss G. C.	Mag.	Right ascension α			Declination δ	Difference between δ 's	$\Sigma\delta$ = sum of declina- tions	$(2\phi - \Sigma\delta)^*$	E-W= $a(2\phi - \Sigma\delta)$	Star north or south, eyepiece east or west	SETTINGS	
		h	m	s							Circle, $\frac{1}{2}$ differ- ence of δ 's	Micro- meter
					° ' "	° ' "	° ' "		turns		° ' "	turns
27910	5.08	20	05	15	23 30					S E	343 31.5	9
28108	4.32	20	12	30	56 27	32 57	79 57	3	1.2	N W	16 28.5	11
28435	6.17	20	24	39	17 11	45 41.5	03	-3	1.2	S W	337 09.5	7
28541	4.28	20	28	57	62 52							
28956	4.63	20	44	25	57 27					N W	342 43.5	6
29268	6.57	20	56	36	22 54	34 33	80 21	-21	8.1	S E	17 16.5	14
29111	6.35	20	50	27	32 42					S E	352 46	7
29327	4.86	20	58	32	47 22	14 40	80 04	-4	1.5	N W	7 20	11
29459	3.92	21	03	33	43 47					N E	356 11	9
29616	6.40	21	09	32	36 09	7 38	79 56	4	1.5	S W	3 49	11
29868	6.81	21	18	27	52 54					N E	347 10.5	8
29996	6.88	21	23	17	27 15	25 39	80 09	-9	3.5	S W	12 44.5	12
30207	4.22	21	32	33	45 25					N E	354 32.5	9
30358	7.22	21	38	27	34 30	10 55	79 55	5	1.9	S	5 27.5	11
30211	6.37	21	32	50	22 35					S E	342 38	8
30222	5.44	21	37	47	57 19	34 44	79 54	-6	2.3	N W	17 22	12

* $(2\phi - \Sigma\delta)$ should not exceed 20 for any pair. The algebraic sum of $(2\phi - \Sigma\delta)$ should not exceed the number of pairs.

TABLE 1 continued
OBSERVING LIST FOR LATITUDEU. S. DEPARTMENT OF COMMERCE
COAST AND GEODETIC SURVEY

Station		Date		Instrument								
OSU FARM		7 AUGUST 1961		WILD T-4								
ϕ : $40^{\circ} 00'$		2ϕ : $80^{\circ} 00'$		Value of one turn: a: 0.387								
Star No. Boss G. C.	Mag.	Right ascension α		Declination δ	Difference between δ 's	$\Sigma\delta$ = sum of declina- tions	$(2\phi - \Sigma\delta)^*$	E-W= $a(2\phi - \Sigma\delta)$	Star north or south, eyepiece east or west	SETTINGS		
		h	m							s	Circle, $\frac{1}{2}$ differ- ence of δ 's	Micro- meter
30483	4.46	21	44	21	50 57				N W	337 05.5	4	
30600	6.85	21	49	53	19 08	41 49	80 05	-5	1.9	S	20 54.5	11
30569	5.32	21	48	20	17-06				S	337 15	6	
30904	5.39	22	03	51	62-36	45 30	79-42	18	7.0	N	22 45	14
31044	3.62	22	09	32	58-01				N W	342 04.5	6	
31120	6.69	22	13	01	22-20	55 41	60-21	-21	8.1	S E	17 50.1	14
32072	6.52	22	58	53	30-53				S E	350-54	9	
32220	5.83	23	06	-01	49-05	18 12	79 58	2	0.8	N	9-06	11
875	5.6	23	11	-28.0	56 -57				N W	347-58	8	
32490	6.8	23	18	-13.0	22-52	34 05	79 49	+11	4.3	S E	17 02	12
1616	5.5	23	32	46.0	40 01				N E	359 59	10	
33093	6.68	23	49	17.0	39 59	02	80 00	0	0	S	01	10
44	6.2	00	02	-48.0	34-26				N	354-18	13	
4	5.0	00	08	-20.0	45-51	11-25	80-17	-17	6.6	S	5-42	7
362	4.5	0	16	14.0	36-34				S E	356-25	6	
608	6.6	0	29	03.0	43-44	07-10	80-18	-18	7.0	N	3-35	14
17	4.52	0	36	32.0	29-06				S	349-11	8	
961	5.0	0	46	33.0	50-45	21-39	79-51	+09	3.5	N	10-49	12

* $(2\phi - \Sigma\delta)$ should not exceed 20 for any pair. The algebraic sum of $(2\phi - \Sigma\delta)$ should not exceed the number of pairs.



TABLE II

Latitude Sheet 4: Mean Right Ascension 1962.0 from 1960.0

Boss GC Star Number	MAG.	R.A. 1960.0			A.V.	S.V.	CORR ⁿ Z(AV)+ .02(SV)	R.A. 1962.0			SET
		h	m	s				h	m	s	
25122	3.69	18	21	46.690	-1.0829	-.0080	-2.1656	18	21	44.524	4
25527	6.36	18	37	55.347	2.9042	.0005	5.8084	18	38	01.155	
25757	5.76	18	45	49.434	1.3408	-.0009	2.6816	18	45	52.116	5
26111	6.70	18	58	22.222	2.4162	.0012	4.8324	18	58	27.055	
26621	3.98	19	16	10.672	1.3861	-.0031	2.7721	19	16	13.444	7
26968	5.96	19	29	43.298	2.4588	.0009	4.9176	19	29	48.216	
27910	5.08	20	05	10.169	2.5776	.0009	5.1552	20	05	15.324	9
28108	4.32	20	12	28.093	1.3942	-.0059	2.7883	20	12	30.881	
28435	6.17	20	24	33.466	2.7404	.0001	5.4808	20	24	38.947	10
28541	4.28	20	28	54.757	1.0059	-.0156	2.0087	20	28	56.768	
28956	4.63	20	44	21.547	1.4890	-.0041	2.9779	20	44	24.525	11
29268	6.57	20	56	31.857	2.6704	.0020	5.3408	20	56	37.198	
29459	3.92	21	03	28.413	2.1821	.0043	4.3643	21	03	32.777	12
29616	6.40	21	09	27.226	2.4136	.0048	4.8273	21	09	32.053	
29868	6.81	21	18	23.475	1.9293	.0036	3.8587	21	18	27.334	13
29996	6.88	21	23	14.062	2.6384	.0042	5.2769	21	23	19.339	
30211	6.37	21	32	44.199	2.7409	.0036	5.4819	21	32	49.681	14 ^b
30372	5.64	21	37	43.040	1.8617	.0042	3.7235	21	37	46.763	
30569	5.32	21	48	14.364	2.8547	.0026	5.7095	21	48	20.073	15 ^b
30904	5.39	22	03	47.415	1.8239	.0061	3.6479	22	03	51.063	
31044	3.62	22	09	27.771	2.0820	.0119	4.1642	22	09	31.935	16
31120	6.69	22	12	55.364	2.8255	.0059	5.6511	22	13	01.015	
32072	6.52	22	58	47.723	2.8635	.0134	5.7273	22	58	53.450	17
32220	5.83	23	05	55.928	2.7258	.0248	5.4521	23	06	01.380	
32329	5.65	23	11	20.807	2.8902	.0386	5.7812	23	11	26.588	18
32490	6.84	23	18	12.603	2.9699	.0111	5.9400	23	18	18.543	
32780	5.50	23	32	39.728	2.9359	.0230	5.8723	23	32	45.600	19
33093	6.68	23	49	16.932	3.0371	.0256	6.0747	23	49	23.007	
44	6.23	00	02	47.539	3.1433	.0234	6.2871	00	02	53.826	20
169	5.08	00	08	13.940	3.1195	.0339	6.2397	00	08	20.180	
362	4.51	00	16	13.852	3.1356	.0256	6.2717	00	16	20.124	21
608	6.64	00	29	03.131	3.2319	.0346	6.4645	00	29	09.595	
759	4.52	00	36	26.136	3.1714	.0212	6.3432	00	36	32.479	22
961	5.03	00	46	33.310	3.4020	.0477	6.8050	00	46	40.115	

TABLE III

LATITUDE SHEET 5 : MEAN DECLINATION 1962.0 FROM 1960.0

BOSS GC STAR NUMBER	DECLINATION 1960.0 ° ' "	A.V. "	S.V. "	CORRECTION 2(AV) + .02(SV)	DECLINATION 1962.0 ° ' "	SET
25122	72 42 57.34	1.556	-.141	3.109	72 43 00.45	4
25527	7 19 16.39	3.202	.416	6.412	7 19 22.80	
25757	52 56 35.52	3.953	.190	7.910	52 56 43.43	5
26111	26 54 30.60	4.988	.339	9.983	26 54 40.58	
26621	53 17 38.19	6.643	.189	13.290	53 17 51.48	7
26968	26 31 52.15	7.641	.328	15.289	26 32 07.44	
27910	23 29 52.68	10.379	.317	20.764	23 30 13.44	9
28108	56 26 41.03	11.016	.167	22.035	56 27 03.07	
28435	17 10 59.99	11.772	.318	23.550	17 11 23.54	10
28541	62 51 33.13	12.100	.112	24.202	62 51 57.33	
28956	57 26 08.32	12.924	.157	25.851	57 26 34.17	11
29268	22 53 26.82	13.928	.274	27.862	22 53 54.68	
29459	43 46 02.91	14.365	.216	28.734	43 46 31.64	12
29616	36 08 05.62	14.706	.233	29.417	36 08 35.04	
29868	52 53 16.93	15.241	.176	30.486	52 53 47.42	13
29996	27 14 41.94	15.514	.237	31.033	27 15 12.97	
30211	22 34 35.07	15.981	.233	31.967	22 35 07.04	14b
30322	57 18 27.44	16.285	.151	32.573	57 19 00.01	
30569	17 05 55.93	16.737	.220	33.478	17 06 29.41	15b
30904	62 35 24.20	17.568	.122	35.138	62 35 59.34	
31044	58 00 12.88	17.748	.134	35.499	58 00 48.38	16
31120	22 19 27.56	17.872	.179	35.748	22 20 03.31	
32072	30 52 04.69	19.326	.103	38.654	30 52 43.34	17
32220	49 04 39.29	19.610	.086	39.222	49 05 18.51	
32329	56 56 50.16	19.882	.086	39.766	56 57 29.93	18
32490	22 52 20.62	19.704	.071	39.409	22 53 00.03	
32780	40 00 56.74	19.853	.043	39.707	40 01 36.45	19
33093	39 58 37.39	19.965	.013	39.930	39 59 17.32	
44	34 26 10.24	20.142	-.013	40.284	34 26 50.52	20
169	45 50 58.97	20.030	-.024	40.060	45 51 39.03	
362	36 33 49.32	19.959	-.040	39.917	36 34 29.24	21
608	43 43 33.03	19.884	-.067	39.767	43 44 12.80	
759	29 05 41.60	19.548	-.080	39.094	29 06 20.69	22
961	50 45 01.68	19.636	-.107	39.270	50 45 40.95	

LATITUDE SHEET 6 : BESSELIAN STAR NUMBERS

BOSS GC STAR NO.	a'	-b'			α'	c'	d'	SET
	$\cos \alpha_0$	$\sin \alpha_0$	$\sin \delta_0$	$\cos \delta_0$				
25122	+.09472	+.99550	.95491	.29690	-.361	+1.07937	+.09045	4
25527	.16513	+.98627	.12746	.99184	-.058	+ .55588	+.02105	
25757	.19881	+.98004	.79806	.60257	-.009	+1.04344	+.15866	5
26111	.25228	+.96765	.45261	.89171	-.027	+ .82467	+.11418	
26621	.32649	+.94520	.80175	.59766	.122	+1.01700	+.26176	7
26968	.38189	+.92421	.44675	.89466	.026	+ .80087	+.17061	
27910	.51973	+.85433	.39881	.91703	.001	+ .73839	+.20727	9
28108	.54652	+.83744	.83341	.55265	.082	+ .93759	+.45548	
28435	.58907	+.80735	.29554	.95533	-.016	+ .90630	+.17409	10
28541	.60511	+.79614	.88994	.45607	-.014	+ .65289	+.53851	
28956	.65740	+.75354	.84285	.53814	-.232	+ .86849	+.55409	11
29268	.69660	+.71746	.38907	.92121	.000	+ .67863	+.27103	
29459	.71796	+.69608	.69183	.72206	.002	+ .79470	+.49671	12
29616	.73590	+.67709	.58980	.80755	-.015	+ .74955	+.43403	
29868	.76169	+.64794	.79755	.60326	-.004	+ .77837	+.60749	13
29996	.77528	+.63162	.45793	.88999	.004	+ .67519	+.35502	
30211	.80080	+.59893	.38406	.92331	-.040	+ .63043	+.30756	14b
30322	.81356	+.58149	.84167	.53999	-.001	+ .72359	+.68475	
30569	.83946	+.54341	.29418	.95575	-.061	+ .57433	+.24695	15b
30904	.87430	+.48538	.88781	.46020	.060	+ .63050	+.77621	
31044	.88607	+.46356	.84817	.52972	.006	+ .62290	+.75154	16
31120	.89301	+.45003	.38001	.92498	-.004	+ .57214	+.33935	
32072	.96466	+.26349	.51322	.85826	.005	+ .50742	+.49508	17
32220	.97239	+.23335	.75572	.65490	.132	+ .46035	+.73485	
32329	.97764	+.21029	.83827	.54525	.299	+ .41273	+.81953	18
32490	.98350	+.18091	.38886	.92130	.001	+ .46988	+.38244	
32780	.99294	+.11858	.64315	.76574	-.042	+ .40834	+.63861	19
33093	.99894	+.04613	.64263	.76618	-.054	+ .36191	+.64195	
44	.99992	-.01264	.56565	.82465	.100	+ .35047	+.56560	20
169	.99934	-.03637	.71765	.69640	-.001	+ .27590	+.71718	
362	.99746	-.07121	.59587	.80308	-.036	+ .30583	+.59436	21
608	.99192	-.12689	.69135	.72252	-.004	+ .22560	+.68576	
759	.98732	-.15877	.48642	.87372	-.249	+ .30167	+.48025	22
961	.97934	-.20223	.77452	.63255	-.004	+ .11768	+.75852	

TABLE V

Latitude Sheet 7 : Besselian Days and Day Numbers

Sets	4,5	7	9,10,11	12,13,14b	15b,16	17,18,19	20,21,22
	h m s	h m s	h m s	h m s	h m s	h m s	h m s
Mean Epoch	18 40 06	19 23 01	20 30 56	21 20 40	22 00 41	23 24 04	00 24 47
Corr'n.	5 32 10	5 32 10	5 32 10	5 32 10	5 32 10	5 32 10	5 32 10
GST	00 12 16	00 55 11	02 03 06	02 52 50	03 32 51	04 56 14	05 56 57
ST, Ch GCT	21 05 03	21 05 03	21 05 03	21 05 03	21 05 03	21 05 03	21 05 03
Interval	03 07 13	03 50 08	04 58 03	05 47 47	06 27 48	07 51 11	08 51 54
G. Day	Aug.8 130011	Aug.8.159815	Aug.8.206979	Aug.8.241516	Aug.8.269306	Aug.8.327210	Aug.8.369375
A	-11.223	-11.226	-11.223	-11.220	-11.218	-11.214	-11.211
B	+ 7.878	+ 7.877	+ 7.875	+ 7.874	+ 7.873	+ 7.871	+ 7.870
C	+13.356	+13.362	+13.373	+13.380	+13.387	+13.399	+15.409
D	-14.386	-14.379	-14.367	-14.359	-14.352	-14.338	-14.328
	- 0.3989	- 0.3989	- 0.3987	- 0.3986	- 0.3985	- 0.3984	- 0.3983

Latitude Sheet 8 : Reduction from Mean to Apparent Declination

Boss GC Star Number	Aa'	Bb'	Cc'	Dd'	$T \mu'$	Sum	Declination		
							°	'	"
25122	- 1.0635	+7.8425	+14.4161	- 1.3012	+.1440	+20.0379	72	43	20.488
25527	- 1.8541	+7.7698	+ 7.4237	- .3028	+.0231	+13.0597	7	19	35.860
25757	- 2.2322	+7.7208	+13.9362	-2.2825	+.0036	+17.1459	52	57	00.576
26111	- 2.8326	+7.6231	+11.0143	-1.6426	+.0108	+14.1730	26	54	54.753
26621	- 3.6652	+7.4453	+13.5892	- 3.7638	-.0487	+13.5568	53	18	05.037
26968	- 4.2871	+7.2800	+10.7012	- 2.4532	-.0104	+11.2305	26	32	18.671
27910	- 5.8329	+6.7278	+ 9.8745	- 2.9778	-.0004	+ 7.7912	23	30	21.231
28108	- 6.1336	+6.5948	+12.5384	- 6.5439	-.0327	+ 6.4230	56	27	09.493
28435	- 6.6111	+6.3579	+ 8.7311	- 2.5012	+.0064	+ 5.9831	17	11	29.523
28541	- 6.7911	+6.2696	+12.1200	- 7.7368	+.0056	+ 3.8673	62	52	01.197
28956	- 7.3780	+5.9341	+11.6143	- 7.9606	+.0925	+ 2.3023	57	26	36.472
29268	- 7.8179	+5.6500	+ 9.0753	- 3.8939	= 0	+ 3.0135	22	53	57.694
29459	- 8.0555	+5.4809	+10.6140	- 7.1323	-.0008	+ .9063	43	46	32.546
29616	- 8.2568	+5.3314	+10.0110	- 6.2322	+.0060	+ .8594	36	08	35.899
29868	- 8.5462	+5.1019	+10.3959	- 8.7229	+.0016	- 1.7697	52	53	45.650
29996	- 8.6986	+4.9734	+ 9.0178	- 5.0977	-.0016	+ .1933	27	15	13.163
30211	- 8.9850	+4.7160	+ 8.4200	- 4.4163	+.0159	- .2494	22	35	06.791
30322	- 9.1281	+4.5787	+ 9.6643	- 9.8323	+.0004	- 4.7170	57	18	55.293
30569	- 9.4171	+4.2783	+ 7.6886	- 3.5442	+.0243	- .9701	17	06	28.440
30904	- 9.8079	+3.8214	+ 8.4405	-11.1402	-.0239	- 8.7101	62	35	50.630
31044	- 9.9399	+3.6496	+ 8.3388	-10.7861	-.0024	- 8.8421	58	00	39.538
31120	-10.0178	+3.5431	+ 7.6592	- 4.8704	+.0016	- 3.6843	22	19	59.626
32072	-10.8177	+2.0739	+ 6.7989	- 7.0985	-.0020	- 9.0454	30	52	34.295
32220	-10.9044	+1.8367	+ 6.1682	-10.5363	-.0526	-13.4884	49	05	05.022
32329	-10.9633	+1.6552	+ 5.5302	-11.7504	-.1191	-15.6474	56	57	14.283
32490	-11.0290	+1.4239	+ 6.2959	- 5.4834	-.0004	- 8.7930	22	52	51.237
32780	-11.1348	+ .9333	+ 5.4713	- 9.1564	+.0167	-13.8699	40	01	22.580
33093	-11.2021	+ .3631	+ 4.8492	- 9.2043	+.0215	-15.1726	39	59	02.147
44	-11.2101	- .0995	+ 4.6995	- 8.1039	-.0398	-14.7538	34	26	35.766
169	-11.2036	- .2862	+ 3.6995	-10.2758	+.0004	-18.0657	45	51	20.964
362	-11.1825	- .5604	+ 4.1009	- 8.5160	+.0143	-16.1437	36	34	13.096
608	-11.1204	- .9986	+ 3.0251	- 9.8256	+.0016	-18.9179	43	43	53.882
759	-11.0688	-1.2495	+ 4.0451	- 6.8810	+.0992	-15.0550	29	06	05.635
961	-10.9794	-1.5916	+ 1.5780	-10.8681	+.0016	-21.8595	50	45	19.091

TABLE VII

Latitude Sheet 9 : Observation Data

BOSS G.C. STAR (SET) NUMBER	N E S W	MICROMETER		LEVEL			CHRONOMETER TIME	
		READING	DIFF ZD	N	S	Diff.	OF OBSERVATION	
		t d	t d	d	d	d	h m s	$\Delta \lambda$ s
25122 (4)	N W	10 27.0	- .939	51.2	24.2	-22.5	18 21 46.0	+0.2
25527	S E	11 20.9		144.5	120.0		18 38 02.5	+0.5
25757 (5)	N E	8 24.2	-3.374	23.2	50.7	-19.3	18 45 55.0	-0.9
26111	S W	11 61.6		116.6	141.2		18 58 28.0	+0.9
26621 (7)	N W	12 36.0	+4.026	49.4	22.2	+0.9	19 16 14.5	+0.8
26968	S E	8 33.4		153.0	128.9		19 29 50.0	+0.1
27910 (9)	S E	9 49.4	+1.208	31.0	58.3	*	20 05 18.0	-1.0
28108	N W	10 70.2		143.8	119.7		20 12 32.5	+0.1
28435 (10)	S W	8 88.5	-1.217	22.3	50.2	-32.4	20 24 40.0	+0.7
28541	N E	10 10.2		126.0	151.2		20 29 00.5	-1.5
28956 (11)	N W	14 26.0	+7.859	43.2	15.4	-37.1	20 44 28.5	-2.1
29268	S E	6 40.1		142.0	116.7		20 56 38.0	+1.0
29459 (12)	N W	10 87.5	+2.140	11.8	41.4	* +16.6	21 03 34.0	0
29616	S E	8 73.5		111.2	136.9		21 09 34.5	-1.3
29868 (13)	N E	11 64.3	-3.307	23.9	51.9	-32.0	21 18 30.5	-1.2
29996	S W	8 33.6		131.2	157.0		21 23 20.5	+0.4
30211 (14b)	S W	10 77.4	+2.588	20.4	48.9	-53.0	21 32 50.5	+0.8
30322	N E	8 18.6		123.9	149.6		21 37 50.0	-1.7
				42.1	14.0	*		
				140.2	114.5			
				48.4	20.2			
				149.0	123.3			
				27.3	55.2	*		
				143.2	168.2			

Reversed

* Second level readings estimated

TABLE VII (Continued)

Latitude Sheet 10 : Observation Data

BOSS GC STAR (SET) NUMBER	N E S W	MICROMETER		LEVEL			CHRONOMETER TIME OF OBSERVATION h m s	$\Delta \lambda$ s
		READING	DIFF ZD	N	S	DIFF.		
		t d	t d	d	d	d		
30569 (15b)	S E	6 91.7	+7.144	19.9	48.2	-26.6	21 48 22.5	-1.0
30904	N W	14 06.1		125.6	151.3		22 03 52.5	-0.1
				92.3	14.1			
				143.9	118.1			
31044 (16)	N W	6 66.2	-7.916	46.8	18.6	-25.9	22 09 33.0	+0.2
31120	S E	14 57.8		151.4	125.6		22 13 03.0	-0.7
				23.0	51.3			
				134.0	160.0			
32072 (17)	S E	9 24.0	+1.113	17.0	45.8	-9.7	22 58 56.0	-1.5
32220	N W	10 35.3		120.2	146.3		23 06 02.0	+0.3
				43.1	14.6			
				144.1	117.8			
32329 (18)	N W	12 11.2	+4.113	48.6	20.0	-20.0	23 11 27.0	+0.7
32490	S E	7 99.9		152.8	126.7		23 18 21.0	-1.5
				24.0	52.6			
				132.8	158.7			
32780 (19)	N E	9 90.1	+.043	18.9	47.5	-23.5	23 32 47.5	-1.1
33093	S W	9 94.4		122.1	148.4		23 49 23.0	+0.9
				42.0	13.5			
				142.1	115.8			
44 (20)	S W	7 29.3	-6.815	48.5	20.5	-0.6	00 02 53.5	-1.3
169	N E	14 10.8		150.5	124.2		00 08 22.0	-0.7
				28.0	56.5			
				143.0	116.8			
362 (21)	S E	14 00.8	-6.883	26.5	55.2	-39.7	00 16 21.5	-0.4
608	N W	7 12.5		136.2	162.5		00 29 09.0	+1.6
				48.1	19.2			
				149.8	123.6			
759 (22)	S W	12 01.2	+3.622	46.7	18.1	-47.0	00 36 32.5	+0.9
961	N E	8 39.0		148.0	122.0		00 46 42.0	-1.1
				26.4	55.2			
				137.0	163.2			

TABLE VIII

LATITUDE SHEET 11 : LATITUDE COMPUTATION

SET	HALF SUM OF DECLINATIONS ° ' "	CORRECTIONS			LATITUDE ° ' "
		MICROMETER ' "	LEVEL "	REFR "	
4	40 01 28.174	- 1 11.721	-2.768	-0.01	40 00 13.675
5	39 55 57.664	+ 4 17.706	-2.374	+0.07	40 00 13.066
7	39 55 11.854	+ 5 7.506	+0.111	+0.09	40 00 19.561
9	39 58 45.362	+ 1 32.267	-3.985	+0.03	40 00 13.674
10	40 01 45.360	- 1 32.954	+2.362	-0.03	40 00 14.738
11	40 10 17.583	-10 00.270	-2.042	-0.19	40 00 15.081
12	39 57 34.222	+ 2 43.453	-4.563	+0.05	40 00 13.162
13	40 04 29.406	- 4 12.589	-3.936	-0.08	40 00 12.801
14b	39 57 01.042	+ 3 17.671	-6.519	+0.06	40 00 12.254
15b	39 51 09.535	+ 9 05.659	-3.272	+0.16	40 00 12.082
16	40 10 19.582	-10 04.624	-3.186	-0.19	40 00 11.942
17	39 58 49.658	+ 1 25.011	-1.193	+0.03	40 00 13.506
18	39 55 02.760	+ 5 14.151	-2.460	+0.10	40 00 14.551
19	40 00 12.364	+ 0 03.284	-2.891	+0.00	40 00 12.757
20	40 08 58.365	- 8 40.530	-0.074	-0.24	40 00 17.521
21	40 09 03.489	- 8 45.724	-4.883	-0.30	40 00 12.582
22	39 55 42.363	+ 4 36.648	-5.781	+0.16	40 00 13.390

TABLE IX

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LATITUDE SHEET 12 : SUMMARY OF LATITUDE COMPUTATIONS

SET	M t	ϕ 40° 00'	$\phi_m - \phi$ $\Delta\phi$ "	Mr "	ϕ "	$\phi_m - \phi$ $\Delta\phi$ "
4	- .939	13.675	- .450	+ .050	13.725	- .510
5	+ 3.374	13.066	+ .159	- .178	12.888	+ .327
7	+ 4.026	19.561	REJECTED (LEVEL READINGS UNCERTAIN)			
9	+ 1.208	13.674	- .449	- .064	13.610	- .395
10	- 1.217	14.738	-1.513	+ .064	14.802	-1.587
11	- 7.859	15.091	REJECTED (LEVEL READINGS UNCERTAIN)			
12	+ 2.140	13.162	+ .063	- .113	13.049	+ .166
13	- 3.307	12.801	+ .424	+ .175	12.976	+ .239
14b	+ 2.588	12.254	REJECTED (LEVEL READINGS UNCERTAIN)			
15b	+ 7.144	12.082	+1.143	- .378	11.704	+1.511
16	- 7.916	11.942	+1.283	+ .419	12.361	+ .854
17	+ 1.113	13.506	- .281	- .059	13.447	- .232
18	+ 4.113	14.551	-1.326	- .217	14.334	-1.119
19	+ .043	12.757	+ .468	- .002	12.755	+ .460
20	- 6.815	17.521	REJECTED ($\Delta\phi$ LARGER THAN 3")			
21	- 6.883	12.582	+ .643	+ .364	12.946	+ .269
22	+ 3.622	13.390	- .165	- .191	13.199	+ .016
+ SUM	+22.757	106.188	+4.183		104.986	+3.842
- SUM	-20.262	65.738	-4.184		66.810	2.256
SUM	+ 2.495	+40.450	- .001		+38.176	+1.586
MEAN	+ .192	13.225			13.215	
SUM (M ²) = 223.074						
SUM ($\Delta\phi^2$) =			8.3518		7.7289	

	UNCORRECTED	CORRECTED
Value of $\frac{1}{2}$ turn of micrometer	76"380	76"327
(1) Mean ϕ , 8 pairs with + M	13.274	13.123
(2) Mean ϕ , 5 pairs with - M	13.148	13.362
Difference, (2) - (1)	- .126	+ .239

$$pc - \sum Mr + \sum \Delta\phi = 0 \quad - \sum Mc + \sum M^2r - \sum M\Delta\phi = 0$$

$$13c - 2.495r - 0.001 = 0 \quad - 2.495c + 223.074r + 11.7699 = 0$$

$$c = -0.01014 \quad r = -0.05278$$

$$e_p = \pm 0.5876 \quad e_0 = \pm 0.146 \quad e_r = \pm 0.0379$$

Latitude of station OSU FARMS : 40° 00' 13"215

Reduction to Sea Level - 0.04

Final Latitude of station : 40° 00' 13"175 $\pm 0"146$

Latitude Sheet 13: Comparison of Chronometer to Radio Signals

Station OSU Farms LAT: 40° 00' 13" LONG: 5^h 32^m 10^s
 State: OHIO Chronometer No.: 2 E 12304 (sidereal)

Local Date	7 Aug 1961	8 Aug 1961	8 Aug 1961	8 Aug 1961	8 Aug 1961
EST Signal	22 20 00.0	00 05 00.0	02 55 00.0	03 40 00.0	04 15 00.0
GCT Date	8 Aug 1961	8 Aug 1961	8 Aug 1961	8 Aug 1961	8 Aug 1961
GCT Time	03 20 00.0	05 05 00.0	07 55 00.00	08 40 00.0	09 15 00.0
ST, 0 ^h GCT	21 05 03.0	21 05 03.0	21 05 03.0	21 05 03.0	21 05 03.0
Correction	32.9	50.1	01 18.0	01 25.4	01 31.2
GCT TO GST	00 25 35.9	02 10 53.1	05 01 21.0	05 46 28.4	06 21 34.2
Longitude	5 32 10.0	5 32 10.0	5 32 10.0	5 32 10.0	5 32 10.0
LST	18 53 25.9	20 38 43.1	23 29 11.0	00 14 18.4	00 49 24.2
Chronometer	18 53 27.0	20 38 45.0	23 29 12.0	00 14 19.5	00 49 25.0
Chron. Corr'n.	-1.145	-1.896	-0.970	-1.077	-0.828
Rate per Minute	-0.007		+0.005	-0.003	+0.007

Remarks: Time Signals from WWV, frequency 5 megacycles

TABLE XI

Latitude Sheet 14 :

Reduction From BOSS G.C. System to FK 3 System

Set	Star	δ	R.A.	$\Delta\delta\delta$	$\Delta\delta\alpha$	Sum	$\frac{\text{Sum}}{z}$
4	25122	72.7	18.3	+.188	+.111	+.607	+.303
	25527	7.3	18.6	+.212	+.096		
5	25757	52.9	18.8	+.180	+.096	+.559	+.279
	26111	26.9	18.9	+.190	+.093		
9	27910	23.5	20.1	+.211	+.059	+.491	+.245
	28108	56.4	20.2	+.144	+.077		
10	28435	17.2	20.4	+.238	+.053	+.453	+.226
	28541	62.8	20.5	+.111	+.051		
12	29495	43.8	21.0	+.197	+.041	+.486	+.243
	29616	36.1	21.2	+.205	+.043		
13	29868	52.9	21.3	+.180	+.044	+.459	+.229
	29996	27.2	21.4	+.190	+.045		
15b	30569	17.1	21.8	+.239	+.049	+.451	+.226
	30904	62.6	22.0	+.112	+.051		
16	31044	58.0	22.2	+.134	+.049	+.438	+.219
	31120	22.3	22.2	+.206	+.049		
17	32072	30.9	22.9	+.189	+.043	+.488	+.244
	32220	49.1	23.1	+.217	+.039		
18	32329	56.9	23.2	+.141	+.036	+.413	+.506
	32490	22.9	23.3	+.203	+.033		
19	32780	40.0	23.5	+.174	+.026	+.391	+.196
	33093	40.0	23.8	+.174	+.017		
21	362	36.6	00.3	+.203	-.014	+.357	+.178
	608	43.7	00.5	+.198	-.030		
22	759	29.1	00.6	+.186	-.038	+.300	+.150
	961	50.8	00.8	+.207	-.055		

Sum +2.945
Mean +0.226

Computed Latitude 40° 00' 13".215
Reduction to FK 3 + 0.226
 40° 00' 13.441
Reduction to Sea Level - .04
Final Latitude 40° 00' 13".401

2.4 Determination of Longitude

2.41 General

The instrument is sighted in the meridian at the zenith distance (Z.D.) obtained in the observing list (Table XII). This ZD., is obtained by using an assumed latitude (ϕ) and the declination (δ) of the star: $Z.D. = \phi - \delta$. When the star enters the field of view, the micrometer movable wire is made to follow the star toward the center (which represents the meridian). Small electrical contacts on the micrometer then send an impulse to the chronograph for each tenth of a turn of the micrometer. These pulses deflect a stylus to make a mark on the chronograph tape, which runs at a constant speed of 1 centimeter per second. The chronometer also is wired to send pulses to the chronograph (on odd seconds) which deflect a second stylus on the other edge of the chronograph tape. Noting the initial micrometer reading and the initial minute on the chronograph, the tapes may be scaled to obtain the sidereal time to the nearest $0^s.01$ for each micrometer pulse, or "break". Ten such breaks are scaled before and after transit of the star. The instrument is plunged and transited before observing the breaks after transit of the star. The method of scaling the tapes is covered in detail in Odermatt [10], pp. 53-55. The mean value of the first and last breaks is the approximate chronometer time of transit. Similarly, the mean of the second and second-from-last, third and third-from-last, etc., will

give a total of ten values for chronometer time of transit. The mean of these ten values gives the chronometer time of transit to the nearest $0^s.001$ for the star. The observed chronometer time of transit ($\text{MEAN}/2$) is corrected by the chronometer correction (Δt), deviation from the set Z.D. (Bb), diurnal aberration (k) and the combined effect of lost motion and average width of contact strips (l). The apparent time of transit ($t = \text{MEAN}/2 + l + k + \text{Bb} + \Delta t$) should then be exactly equal to the apparent R.A. (α) of the star, which in turn should be exactly equal to the local sidereal time (L.S.T.). The difference ($\alpha - t$) thus is a differential correction to the assumed longitude used in Table XVII to compute chronometer corrections.

2.42 The Observing List

The observing list (Table XII) is made up from the Apparent Places [2] in accordance with the criteria listed in [8] p. 37. The column headed A is the azimuth factor for each star. A was computed to five decimals by the formula:

$$A = \sin \phi - \tan \delta \cos \phi$$

Where: ϕ = assumed latitude = $40^{\circ}00' 00''00$

δ = declination of the star

The column headed B is the level factor computed to five decimals by the formula:

$$B = \cos \phi + \tan \delta \sin \phi$$

Where: ϕ and δ are as defined above

The column headed $\sec \delta$ is the secant of the declination to five decimals. Although this value is listed in the Apparent Places [2] for each star, it is only carried to three places. The column headed k has been computed by the formula:

$$k = 0^s.021 \cos \phi \sec \delta$$

Where ϕ and δ are as defined above

The factor k is the correction for diurnal aberration, or, the sighting error caused by the rotation of the earth. The derivation of this factor is covered in detail in Nassau [9] pp. 159, 179, 191-192. A brief derivation follows:

Because of the finite velocity of light, the meridian of the observer will move eastward an amount K during the time it takes light to travel from the star to the observer.

$$K = \frac{v}{c \sin 1''} = 0''.319 \cos \phi = 0^s.021 \cos \phi$$

Where: $v = \text{speed of observer} = \frac{2\pi R \cos \phi}{24.60.60}$ miles/second

$$R = \text{mean radius of earth} = 3959 \text{ miles}$$

$$\phi = \text{latitude of observer}$$

$$c = \text{velocity of light} = 186,300 \text{ miles/second}$$

Thus, at the instant the star appears to be on the meridian in the instrument it has already transited the celestial meridian by a small angle k . By the law of sines, then:

$$\frac{\sin k}{\sin K} = \frac{\sin 90^\circ}{\cos \delta} = \sec \delta$$

Where: $\delta = \text{declination of the star}$

Since k and K are small angles we may say that the sine of the angle is equal to the angle itself: $k = K \sec \delta$

Substituting the value obtained for K, we obtain:

$$k = 0^s.021 \cos \phi \sec \delta$$

This correction is subtracted from the observed time of the transit for stars at upper transit (as all stars were in this computation), and added to those observed at lower transit.

The factor l is the correction to be added for the combined effects of the lost motion ($m = 0^p.031$) and width of contact strips ($s = 1^p.114$) and is computed by the formula indicated in [8] p. 26:

$$l = \frac{R}{2} \frac{(m+s)}{100} \sec \delta = 0^s.0583 \sec \delta$$

Where R = equatorial value of one turn of the micrometer
 $= 10^s.1849$

m = lost motion = $0^p.031$

s = average width of contact strips = $1^p.114$

δ = declination of the star being observed

$1^p = 1/100 R$

It is apparent that declination is the only variable in this formula. The constant represents the premature closing of the electrical contact (in seconds of time) for an equatorial star. Multiplying by the $\sec \delta$ corrects R from its equatorial value to its value at that declination. This correction is discussed on Odermatt [10] pp. 51, 63.

2.43 Computation of Chronometer Corrections

The computations in Table XIV are in the format described in reference [8] pp. 125-126. At the top of each sheet is

listed the value of factors $d\psi$ and $d\epsilon$ and the mean epoch of each set of six stars. The mean epoch is the mean value of the six R.A. values in the set converted to a Besselian Day in the same manner described in part 2.34. The values of $d\psi$ and $d\epsilon$ are interpolated from the Apparent Places [2], Table I, p. 479 for each mean epoch.

For each star, the level readings are copied on lines 2 and 3. The difference of the 2 West readings and the difference of the 2 East readings are written on line 4. The difference of the values on line 4 is written on line 5 in the sense West minus East. Line 6 is the value of the apparent inclination of the telescope axis (b) obtained from the level readings:

$$b = \frac{d''}{60} (\Delta L) = 0^s.01495 (\Delta L)$$

Where: d'' = value of division of the hanging level = $0''.89716$

ΔL = the difference of the level readings from line 5

The conversion of d from arc to time includes a factor of $(4) \times (15'' \text{ per } 1^s) = 60''/1^s$ because ΔL is obtained from 4 level readings. On line 7 is copied the level factor (B) from column 8 of Table XII. The product Bb is next written on line 26.

The chronometer time of transit to the nearest whole minute is written on line 8. The whole minutes for the initial and final micrometer breaks are written on line 9 above columns 1 and 2, respectively, of lines 10 through 20. The

ten micrometer breaks before and after transit are written in column 1 from top to bottom and in column 2 from bottom to top, respectively, on lines 11 through 20. The sum of columns 1 and 2 are recorded in column 3, lines 11 through 20, and the mean value of these sums is written on line 21. The data for columns 1 and 2, lines 9 through 20, are taken from the chronograph tapes as described by Odermatt [10], pp. 53-55.

Line 22 is one-half the value of line 21 which is added to the initial minutes on line 9 (column 1) to obtain the chronometer time of transit ($\text{MEAN}/2$) in seconds. Although I did not rescale the chronograph tapes, I found it necessary to examine them carefully to verify the listed figures and account for the 30 second difference between lines 22 and 23 in certain cases. For this purpose, I found it expedient to lay out a line 3 minutes long on the scale: 1 second = 1 millimeter. The measured mid-point between the first and last breaks and the middle two breaks (line 20, columns 1 and 2) each gave a simple check on the time of transit.

The values l and k are copied onto lines 24 and 25 from Table XII, columns 10 and 11, respectively, for each star. Line 27 is the algebraic sum of line 8 and 23 through 26, and is the chronometer time of transit corrected for the inclination (Bb) lost motion and width of contact strips (l), and diurnal aberration (k).

The R.A. of the star is interpolated for the epoch of observation from the Apparent Places [2] by using second differences and Bessel's formula as explained in the Apparent Places [2], p. IX (Also see Nassau [9], p.245 and Nautical Almanac [1] p. 468):

$$\alpha = \alpha_0 + n (\Delta'_{\frac{1}{2}}) + B''n (\Delta''_0 + \Delta''_1)$$

Where: α = apparent R.A. for desired epoch

α_0 = apparent R.A. for nearest whole day (X)

$$n = \text{interpolation factor} = \frac{(\text{DESIRED EPOCH}) - X}{10}$$

$\Delta'_{\frac{1}{2}}$ = first difference

$B''n (\Delta''_0 + \Delta''_1)$ = second difference correction, which is obtained directly from [2], Table VI

This apparent R.A. is written on line 28.

The correction for short period terms in nutation ($\Delta\alpha$) on line 29 is obtained by the formula listed in the Apparent Places [2] p. IX (also see Nassau [9] p. 163):

$$\Delta\alpha = d\alpha(\psi) d\psi + d\alpha(\epsilon) d\epsilon$$

Where: $\Delta\alpha$ = short period term

$d\alpha(\psi)$ = change in α due to change in ψ

$d\alpha(\epsilon)$ = change in α due to change in ϵ

$d\psi$ = short period term for nutation in longitude

$d\epsilon$ = short period term for nutation in obliquity

The values for $d\psi$ and $d\epsilon$ are listed above each set as previously explained. The values for $d\alpha(\psi)$ and $d\alpha(\epsilon)$ are listed beneath each star in the Apparent Places [2].

The chronometer correction (Δt) on line 30 is obtained by interpolation for each star from the values obtained in

Table XVII.

Finally, the quantity $(\alpha + \Delta\alpha - t - \Delta t)$ on line 31 represents the difference between the R.A. computed for our assumed longitude and the actual L.S.T. at time of transit. These values are now copied into column 2 Table XV.

2.44 Longitude Computation

The values of A and $(\alpha + \Delta\alpha - t - \Delta t)$ for each star are entered in columns 2 and 3, respectively, of Table XV. Column 3 lists the square of each A factor and column 4 the product of columns 2 and 3. Each column 2 through 4 is added and two normal equations are written for each set as described in [8] p. 127:

$$n \Delta\lambda_a - [A]a + [(\alpha + \Delta\alpha - t - \Delta t)] = 0$$

and

$$- [A] \Delta\lambda_a + [AA]a - [A(\alpha + \Delta\alpha - t - \Delta t)] = 0$$

Where: n = number of stars in the set

$\Delta\lambda_a$ = adjusted correction to assumed longitude
for the set

[AA] = the sum of the squares of the A factors

a = correction to the aximuth of the collimation
axis of the instrument

$[(\alpha + \Delta\alpha - t - \Delta t)]$ = the sum of column 3

$[A(\alpha + \Delta\alpha - t - \Delta t)]$ = the sum of column 5
[A] = the sum of column 2

These normal equations are solved by the method of least squares to obtain the values of a and $\Delta\lambda_a$. The products of a with each A are now entered in column 6; the difference

$Aa - (\alpha + \Delta\alpha - t - \Delta t)$ gives the correction to the assumed longitude for each star, and is entered in column 7 ($\Delta\lambda$). The residual (v) for each star is then obtained by:

$$\Delta\lambda_a - \Delta\lambda = v$$

Any star having a residual greater than $0^s.20$ was rejected (see [8], p. 123) and a second adjustment made with the remaining stars. In the case of set 1, only one star fell within the limits and the entire set was rejected. The probable explanation is that the observers were completely inexperienced with both the instrument and the method. Although the Longitude Record, Table XIII, does not indicate it, Dr. Mueller stated that considerable difficulty was experienced with the chronograph during this first set.

Finally, the mean of the $\Delta\lambda_a$ values obtained was applied to the assumed longitude, and the result taken as the mean observed longitude. The value obtained is $83^{\circ} 02'28''.230 \pm 5''.460$.

2.45 Radio Chronometer Comparisons

Before and after the observation of each set of stars, the micrometer was disconnected from the chronograph and the radio connected in its place (a schematic wiring diagram is illustrated in reference [8], figure 8, p. 12). With the radio tuned to WWV continuous time service, the chronograph tape thus records odd seconds from the chronometer pulses, and every second from the time signal. Twenty radio breaks

are scaled in the same manner as the micrometer breaks are scaled, and the results listed in columns 2 and 3 of Table XVI. The correction in column 3 is obtained directly from reference [8], p. 193 (Table XV), which reduces the breaks to the whole second nearest the mean epoch of the break. The mean values of the radio time and the chronometer time give an accurate comparison for the determination of the chronometer correction and rate. These values are entered onto Table XVII for the computation of chronometer correction and rate in the same manner as Table X.

WWV time signals are monitored by the Naval Observatory in Washington, D.C., which computes and publishes corrections to the broadcast time signals at a later date. This correction has been omitted in these computations as the data were not received in time. It is unlikely that this correction would be greater than $\pm 0^s.01$.

Twenty seconds before a time check a voice announcement is made of the Eastern Standard Time followed by Morse Code signal of the Universal Time, a second voice announcement and two warning pulses. The tone deflects the stylus indicating the zero second of the announced time, and is broken every second thereafter except the fifty-ninth, which is always omitted to identify the beginning of succeeding minutes. Figure 7 illustrates the appearance of the signal in the chronograph tape (illustrating signal at 1005 EST, or 1505 UT).

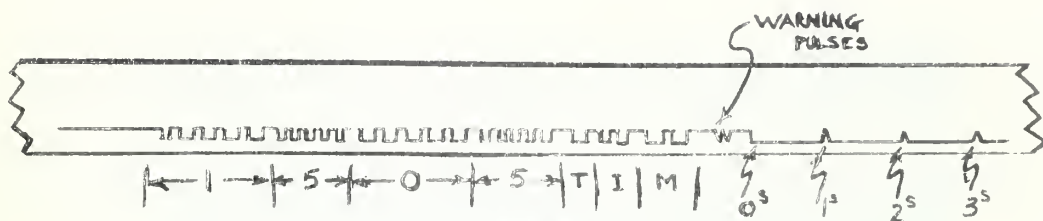


FIGURE 7

The pulses between the last code group of Universal Time and the two warning pulses are not explained in the literature, but after running thirty time checks on the chronograph, these unexplained pulses took the general form shown in Figure 7 in more than two-thirds of the checks made. The form of the pulses thus appears to be the Morse Code letters " T I M ". Whatever they are, they are distinguished from the beginning of the time tone as being individual pulses, while the tone is a continuous pulse broken very briefly each second. The two warning pulses immediately before the tone are of only $0^s.005$ duration spaced $0^s.1$ apart, and are not always indicated on the tape.

The time signals are given continuously in five minute cycles, except a silent period 45 to 49 minutes after each hour. Each cycle begins with a 600 cps tone for two minutes, a one minute period of rapid code, and finally two minutes of a 440 cps tone. Details of the time service may be found in reference [14].

TABLE XII

Longitude Observing List $Q = 40^{\circ} 00' 00'' 000$ $\sin Q = 0.6427$ 8761 $\cos Q = 0.7660$ 4444

Cat.	Star	Mag.	α h m s	δ	Z.D.	A.	B	sec δ	$k(-)$	$\ell(+)$
733	ζ Cygni	3.9	19 28 48	51° 39' 15"	-11° 38' 9"	-0.32545	1.57849	+1.61170	0.02593	0.09396
738	η Cygni	4.6	19 35 24	50° 08' 15"	-10 07.9	-0.27447	1.53572	+1.56006	0.02510	0.09095
1521	λ Cygni	4.0	19 54 54	34° 59' 0"	5 01.2	+0.10674	1.21585	+1.22052	0.01963	0.07116
757	ϕ Cygni	4.0	20 12 24	46° 37' 6"	- 6 37.4	-0.16803	1.44641	+1.45613	0.02343	0.08489
1535	42 Cygni	5.9	20 27 54	36° 19' 7"	3 40.5	+0.07950	1.23870	+1.24124	0.01997	0.07236
792	ϵ Cygni	3.9	21 03 36	43° 46' 5"	3 46.3	-0.09118	1.38192	+1.38492	0.02228	0.08074
1558	ζ Cygni	4.3	21 15 54	39° 14' 1"	0 46.1	+0.01724	1.29094	+1.29105	0.02077	0.07527
807	71 Cygni	5.3	21 28 06	46° 22' 3"	6 22.1	-0.16083	1.44037	+1.44931	0.02331	0.08449
1575	14 Pegasi	5.0	21 48 12	29° 59' 7"	10 00.5	+0.20060	1.13708	+1.15464	0.01858	0.06732
835	π Pegasi	4.4	22 08 18	32° 59' 4"	7 00.8	+0.14551	1.18331	+1.19223	0.01918	0.06951
1590	38 Pegasi	5.5	22 28 18	32° 22' 5"	7 37.7	+0.15711	1.17358	+1.18404	0.01905	0.06903
844	ρ Lacertae	4.6	22 22 06	52° 02' 1"	12 01.9	- 0.33893	1.58981	+1.62554	0.02615	0.09477
1600	ϵ Lacertae	6.0	22 53 18	36° 52' 3"	3 07.9	+0.06822	1.24816	+1.25003	0.02011	0.07288
869	σ Andromedae	3.6	23 00 12	42° 07' 1"	- 2 06.9	-0.04983	1.34722	+1.34814	0.02169	0.07860
1610	12 Andromedae	5.8	23 19 00	32° 58' 3"	2 01.9	+0.04490	1.26773	+1.26852	0.02040	0.07395
1616	15 Andromedae	5.5	23 32 48	40° 01' 4"	- 0 01.2	-0.00052	1.30585	+1.30585	0.02101	0.07613
1619	χ Andromedae	4.3	23 38 30	44° 02' 1"	- 4 06.9	-0.10003	1.38935	+1.39295	0.02241	0.08121
1629	ψ Pegasi	4.8	23 55 30	24° 55' 6"	15 04.6	+0.28677	1.06478	+1.10275	0.01775	0.06429
4	22 Andromedae	5.1	0 08 18	45° 51' 4"	- 5 51.2	-0.14651	1.42835	+1.43583	0.02311	0.08371
1005	ϕ Andromedae	4.5	0 16 18	36° 34' 2"	3 26.0	+0.07450	1.24289	+1.24512	0.02003	0.07259
17	σ Cassiopeiae	3.7	0 34 48	53° 40' 9"	-13 40.7	-0.39935	1.64050	+1.68841	0.02716	0.09843
25	ϕ Cassiopeiae	4.7	0 42 36	48° 04' 2"	- 8 04.0	-0.21007	1.48168	+1.49651	0.02408	0.08725
33	μ Andromedae	3.9	0 54 36	38° 17' 4"	+1 42.8	+0.03802	1.27350	+1.27408	0.02050	0.07428
1030	μ Cassiopeiae	5.3	1 05 42	54° 43' 7"	-14 43.5	-0.44026	1.67484	+1.73175	0.02786	0.10096

TABLE XII (Continued)

Cat.	Star	Mag.	α h m s	δ	μ .D.	A	B	sec δ	$k(-)$	$\ell(+)$
1521	η Cygni	4.0	19 54 54	34° 59.0	5 01.2	+0.10674	1.21585	+1.22052	0.01963	0.07116
1535	42 Cygni	5.9	20 27 54	36° 19.7	3 40.5	+0.07950	1.23870	+1.24124	0.01997	0.07236
792	ξ Cygni	3.9	21 03 18	43° 46.5	- 3 46.3	-0.09118	1.38192	+1.38492	0.02228	0.08074
1558	ζ Cygni	4.3	21 15 54	39° 14.1	0 46.1	+0.01724	1.29094	+1.29105	0.02077	0.07527
1575	14 Pegasi	5.0	21 48 12	29° 59.7	10.00.5	+0.20061	1.13708	+1.15464	0.01858	0.06732
844	β Lacertae	4.6	22 22 01	52° 02.1	-12 01.9	-0.33893	1.58981	+1.62554	0.02615	0.09477
835	π Pegasi	4.4	22 08 18	32° 59.4	7 00.8	+0.14551	1.18331	+1.19222	0.01918	0.06951
1590	38 Pegasi	5.5	22 28 18	32° 22.5	7 27.7	+0.15711	1.17358	+1.18404	0.01905	0.06903
852	10 Lacertae	4.9	22 37 36	38° 51.0	1 09.2	+0.02578	1.28378	+1.28404	0.02066	0.07486
858	13 Lacertae	5.2	22 42 00	41° 37.0	- 1 36.8	-0.03773	1.33707	+1.33761	0.02152	0.07798
869	ϵ Andromedae	3.6	23 00 12	42° 07.1	- 2 06.9	-0.04983	1.34722	1.34814	0.02169	0.07860
1610	12 Andromedae	5.8	23 19 00	37° 58.3	2 01.9	+0.04490	1.26773	1.26852	0.02040	0.07395

TABLE XIII
LONGITUDE RECORD

ORIGINAL TRANSIT LEVEL READINGS

Station: 056

Chief of party: Dr. Muller

Temperature: 65° F

State: Ohio

Observer: Anderson (set I)
(set II)

Barometer:

Local date: 6 Aug '61

Instrument: T-4

Level "C" at Circle Side

SET I				SET II			
STARS (or signal*)		LEVELS		STARS (or signal*)		LEVELS	
		W.	E.			W.	E.
N 733 Cygni (14) 27 ^m 41.5		72.2	31.1	S 1558 Cygni (14) 15 ^m 04.0		71.2	27.0
(14) 29 53.0		32.5	76.0	(14) 16 51.0		34.8	79.1
N 738 Cygni (8 ^b) 34 25.0		32.5	76.0	807 Cygni (8 ^b) 27 06.5		34.9	79.1
(8 ^b) 36 30.5		72.5	23.5	N (6) 29 03.5		71.6	26.9
S 1521 Cygni (14) 54 04.0		73.0	29.0	211 Cygni (14) 34 33.4		71.2	27.9
(14) 55 41.0		33.0	71.0	N (14) <u>Lost</u>		Lost	
N 757 Cygni (6 ^x) 11 30.0		35.0	74.0	1573 Pegasus (6) 47 25.0		34.1	79.0
(6 ^x) 13 27.0		72.1	28.0	S (6) 48 58.0		71.0	26.3
S 1535 Cygni (14) 27 04.0		72.6	28.1	835 Pegasus (14) 07 30.0		72.0	27.1
(14) 28 47.0		33.9	78.1	S (14) 09 09.0		34.1	79.0
N 788 Cygni (6) —		35.2	80.6	844 Lacerte (6) 21 01.0		35.0	80.1
(6)		Lost	—	N (6) 24 11.0		71.1	26.0
N 792 Cygni (6) 02 40.0		35.9	80.5	1590 Pegasus (14) 27 30.0		71.5	26.4
(6) 04 31.0		71.2	26.8	S (14) 29 08.0		33.9	79.1

Chronometer-chronograph correspondences:

EST 20^h 10^m EST 01^h 35^m
Chrono. 16^h 39^m 09.0 Chrono. 22^h 05 32.5

Remarks: #757 - Very bright star also seen off vertical wire.
Trip Signal on Chronometer on 13-14-15

* In proper sequence with stars to show time of reception, include following data for each time signal received: Time (local standard or war); radio-station identification; frequency.

TAELE XIII continued

DEPARTMENT OF COMMERCE
COAST AND GEODETIC SURVEY
Form 34A
Rev. May 1945

LONGITUDE RECORD

ORIGINAL TRANSIT LEVEL READING

Station: OSU

Chief of party:

Temperature:

State: OHIO

Observer:

Barometer:

Local date: 7 AUG '61

Instrument: T-4

Level 0 at circle side

SET I					SET II				
STARS (or signal*)			LEVELS		STARS (or signal*)			LEVELS	
			W.	E.				W.	E.
S	#1600 (14)	12 ^m 26.5	68.5	22.9	#14 (14)	17 ^m 22.0	69.7	23.0	
	Andromedae (6.0) (14)	14 19.5 01.5	30.1	75.9	N Andromedae (14)	19 ^m 26.8	32.2	78.0	
					NEW TAPE				
N	#869 (6)	59 ^m 17.0	33.9	79.8	#1005 (6)	15 ^m 31.0	32.0	78.0	
	Andromedae (6)	01 ^m 05.5	68.2	22.3	S Cassiopeia (6)	17 ^m 11.0	69.2	23.6	
S	#1610 (14)	18 ^m 12.5	70.8	25.0	#17 (14)	33 ^m 40.0	68.9	22.9	
	Andromedae (14)	19 ^m 57.0	32.0	77.5	N Cassiopeia (14)	36 ^m 02.2	32.0	78.0	
N	#1616 (6)	31 ^m 55.2	33.0	78.2	#25 (6)	41 ^m 32.7	32.0	78.1	
	Andromedae (6)	33 39.0	69.6	23.9	Cassiopeia (6)	43 ^m 36.5	69.0	23.0	
N8	#1619 (14)	37 ^m 35.0	69.6	23.9	#33 (14)	53 ^m 46.0	69.0	23.0	
	Andromedae (14)	39 ^m 30.8	32.0	77.8	S Andromedae (14)	55 ^m 31.5	30.8	77.0	
S	#1629 (6)	55 ^m 07.5	31.5	77.0	#1030 (6)	04 ^m 34.5	32.0	78.0	
	Pegasi (6)	56 ^m 34.5	69.4	23.9	N (6)	06 ^m 53.0	69.1	23.0	

Chronometer-chronograph correspondences:

Remarks: EST 0255

CHRON 23^m 20^m 15.8
A P

Triple Signal on Chronometer on 13-14-15

EST 0325
CHRON 23^m 55^m 20.5 +1.4EST 0430
CHRON 01^m 00^m 31.0

* In proper sequence with stars to show time of reception. Include following data for each time signal received: Time (local standard or war); radio-station identification; frequency.

TAELE XIII continued

LONGITUDE RECORD

DEPARTMENT OF COMMERCE
COAST AND GEODETIC SURVEY
Form 31A
Rev. May 1945

ORIGINAL TRA

Station: 054

Chief of party: Mr. Kiviaga

Temperature:

State: Ohio

Observer: Osborne

Barometer:

Local date: 8 Aug. 61

Instrument: T-4

Level 0° on Vertical Circle End.

SET I				SET II			
STARS (or signal*)		LEVELS		STARS (or signal*)		LEVELS	
		W.	E.			W.	E.
1521 Cygni (14) 19 ^h 54 ^m 6.5		68.2	27.0	835 Pegasus (14) 22 ^h 06 ^m 45.5		68.9	25.0 *
S. (14) 55 ^m 46.5		39.7	82.0	(14) 22 ^h 09 ^m 09.5		40.0	83.8
				S			
757 Cygni (6) 11 29.0		41.2	84.0	1590 Pegasus (14) 27 30.0		71.0	25.0
N (6) — —		68.7	25.8	(14) 29 08.0		40.0	83.1
				S			
1535 Cygni (14) 20 ^h 27 ^m 04.5		69.5	26.2	852 Lacertae (6) 36 43.0		40.2	83.4
S (14) 20 28 46.5		39.8	82.8	(6) 38 27.0		69.0	25.2
				S			
788 Cygni (6) — —		41.0	84.0	858 Lacertae (14) 41 30.5		70.8	25.0
N (6) — —				(14) 38 ²³ 20.5		40.0	84.0
				N			
792 Cygni (14) 21 ^h 02 ^m 37.5		68.5	25.0	1600 Lacertae (6)		40.5	84.3
(14) 21 ^h 04 31.5		41.0	83.0	(6) <u>Lost</u>			
N				S			
1558 Cygni (6) 15 05.0		41.0	84.1	869 Andromedae (6) 22 ^h 59 ^m 18.0		41.0	85.0
(6) 16 49.5		68.8	25.4	(6) 23 ^h 01 07.5		68.8	24.8
S				N			
807 Cygni (14) 27 06.0		68.8	25.2	1610 Andromedae (14) 18 12.0		68.8	24.7
(14) 39 05.0		—	—	(14) 19 57.0		40.0	84.2
N				S			
844 1575 Cygni (6) 47 24.5		40.5	84.2				
(6) 49 58.2		68.8	25.0				
N/S							

Chronometer-chronograph correspondences:

Chrono: 21^h 37^m 49.5 EST = 01^h 00^m

Chrono: 23^h 23^m 07.5 EST 02^h 45^m

835 Pegasus (14) 22 ^h 21 00.0	40.5	84.5
13 Lacertae (14) 23 10.5	69.0	25.0
N (6)		

Remarks:

1521 Drum moved prematurely on circle E.

757 " " " " " " W.

807 Drum not recorded.

835 1st Drum wrong because tracking switched mid-way to other star.

* Level estimated.

* In proper sequence with stars to show time of reception, include following data for each time signal received: Time (local standard or war); radio station identification; frequency.



TABLE XIV

Computation of Chronometer Corrections: Sheet 1

Set 1: 6 August 1961 $d\psi = 0.0136$ $d\epsilon = -0.0646$

Star: 733 ζ Cygni			738 γ Cygni			1521 η Cygni			757 ϕ^2 Cygni		
Level: W 72.2 E 31.1			W 32.5 E 76.0			W 73.0 E 29.0			W 35.0 E 79.0		
+ 39.7 - 44.9			+ 40.0 - 47.5			+ 40.0 - 48.0			+ 37.1 - 51.0		
-5.2			-7.5			-8.0			-13.9		
-0.07774			-0.11212			-0.11960			-0.20780		
1.57849			1.53572			1.21585			1.44641		
19 h. 28 m. 27m			19 h. 35 m. 34m			19 h. 54 m. 54m			20 h. 12 m. 11m		
S	s	Sums	S	s	Sums	S	s	Sums	S	s	Sums
36.93	56.14	93.07	23.27	32.27	55.54	2.14	47.64	49.78	27.16	30.70	57.86
39.23	55.00	94.23	26.35	30.06	56.91	3.31	46.08	49.39	28.79	28.76	57.55
41.42	53.58	95.00	27.46	28.78	56.24	4.29	44.91	49.20	30.35	27.38	57.73
42.81	51.74	93.55	28.60	27.03	55.63	5.79	43.60	49.39	31.90	25.50	57.40
44.24	50.03	94.27	29.92	25.20	55.12	6.95	42.64	49.59	33.02	24.68	57.70
45.45	48.29	93.74	31.32	23.80	55.12	8.15	41.34	49.49	34.21	22.74	56.95
46.51	46.18	92.69	32.70	22.07	54.77	9.09	39.98	49.07	35.80	20.76	56.56
48.50	45.48	93.98	34.30	20.61	54.91	10.38	38.77	49.15	37.42	19.30	56.72
50.00	43.41	93.41	38.20	19.06	57.26	11.50	37.49	48.99	38.81	17.25	56.06
51.48	40.96	92.44	39.90	17.01	56.91	12.81	35.70	48.51	40.45	16.26	56.71
Mean		93.638	Mean		55.841	Mean		49.256	Mean		57.124
Mean/2		46.819	Mean/2		27.921	Mean/2		24.628	Mean/2		28.562
		+ 0.094			+ 0.091			+ 0.071			+ 0.085
		- 0.026			- 0.025			- 0.020			- 0.023
		- 0.123			- 0.172			- 0.145			- 0.301
Bb	19h 28m	46.764	19h 35m		27.814	19h 54m		53.631	20h 12m		28.326
t	19h 28m	46.197	19h 35m		26.521	19h 54m		53.193	20h 12m		27.119
α		+ 0.002			+ 0.002			+ 0.002			+ 0.003
$\Delta \alpha$		- 0.897			- 0.899			- 0.903			- 0.907
$\Delta \alpha + \Delta \alpha - \Delta \alpha$		+ 0.882			- 0.393			+ 0.467			- 0.297

TABLE XIV (Continued)

Computation of Chronometer Corrections: Sheet 2

Set 1 (Continued)										Set 2: 6 August 1961									
Star: 1535 42 Cygni					792 ♂ Cygni					1558 ♂ Cygni					807 7/ Cygni				
Level: W 72.6 E 28.1					W 35.9 E 80.5					W 71.2 E 27.0					W 34.9 E 79.1				
+ 38.7 - 50.0					+ 35.3 - 53.7					+ 36.4 - 52.1					+ 36.7 - 52.2				
- 11.3					- 18.4					- 15.7					- 15.5				
-0.16894					-0.27508					-0.23472					-0.23172				
1.23870					1.38192					1.29094					1.44037				
20h 27m					21h 03m					21h 15m					21h 28m				
27 28					02 04					15 16					27 28				
s	s	Sums	s	Sums	s	s	Sums	s	Sums	s	s	Sums	s	s	s	s	Sums	s	Sums
1.22	49.40	50.62	39.86	30.63	70.49	4.40	51.13	55.53	7.06	63.24	70.30	7.06	63.24	70.30	7.06	63.24	70.30	7.06	63.24
2.71	48.19	50.90	41.28	29.20	70.48	5.83	49.64	55.47	8.29	61.87	70.15	8.29	61.87	70.15	8.29	61.87	70.15	8.29	61.87
4.80	46.69	51.49	42.42	27.71	70.13	6.91	48.41	55.32	9.63	60.21	69.84	9.63	60.21	69.84	9.63	60.21	69.84	9.63	60.21
6.08	45.46	51.54	44.02	26.69	70.71	8.22	46.84	55.06	11.01	58.68	69.69	11.01	58.68	69.69	11.01	58.68	69.69	11.01	58.68
7.01	44.19	51.20	45.40	24.88	70.28	9.27	45.31	54.58	12.34	57.45	69.79	12.34	57.45	69.79	12.34	57.45	69.79	12.34	57.45
8.01	42.96	50.97	46.50	23.50	70.00	10.51	44.08	54.59	13.97	55.85	69.82	13.97	55.85	69.82	13.97	55.85	69.82	13.97	55.85
8.91	41.41	50.32	48.13	22.20	70.23	12.10	42.40	54.50	15.32	54.17	69.49	15.32	54.17	69.49	15.32	54.17	69.49	15.32	54.17
10.19	40.25	50.44	49.50	20.60	70.19	13.50	41.45	54.95	17.40	52.33	69.73	17.40	52.33	69.73	17.40	52.33	69.73	17.40	52.33
11.18	38.94	50.12	50.70	19.30	70.00	14.48	40.18	54.56	19.29	50.98	70.27	19.29	50.98	70.27	19.29	50.98	70.27	19.29	50.98
12.28	37.61	49.89	52.20	17.78	69.98	15.80	38.40	54.20	20.73	49.80	70.53	20.73	49.80	70.53	20.73	49.80	70.53	20.73	49.80
Mean	Mean	50.749	Mean	Mean	70.249	Mean	Mean	54.876	Mean	Mean	69.961	Mean	Mean	69.961	Mean	Mean	69.961	Mean	Mean
Mean/2	Mean/2	25.375	Mean/2	Mean/2	35.125	Mean/2	Mean/2	27.438	Mean/2	Mean/2	34.981	Mean/2	Mean/2	34.981	Mean/2	Mean/2	34.981	Mean/2	Mean/2
l	l	+ 0.072	l	l	+ 0.081	l	l	+ 0.075	l	l	+ 0.084	l	l	+ 0.084	l	l	+ 0.084	l	l
k	k	- 0.020	k	k	- 0.022	k	k	- 0.021	k	k	- 0.023	k	k	- 0.023	k	k	- 0.023	k	k
Bb	Bb	- 0.209	Bb	Bb	- 0.380	Bb	Bb	- 0.303	Bb	Bb	- 0.334	Bb	Bb	- 0.334	Bb	Bb	- 0.334	Bb	Bb
t	t	55.218	t	t	34.804	t	t	57.189	t	t	04.708	t	t	04.708	t	t	04.708	t	t
α	α	53.884	α	α	33.724	α	α	55.989	α	α	03.711	α	α	03.711	α	α	03.711	α	α
Δα	Δα	+ 0.002	Δα	Δα	+ 0.004	Δα	Δα	+ 0.004	Δα	Δα	+ 0.004	Δα	Δα	+ 0.004	Δα	Δα	+ 0.004	Δα	Δα
Δα _t	Δα _t	- 0.910	Δα _t	Δα _t	- 0.912	Δα _t	Δα _t	- 0.912	Δα _t	Δα _t	- 0.911	Δα _t	Δα _t	- 0.911	Δα _t	Δα _t	- 0.911	Δα _t	Δα _t
Δα _t + Δα - t - Δα	Δα _t + Δα - t - Δα	- .422	Δα _t + Δα - t - Δα	Δα _t + Δα - t - Δα	- .164	Δα _t + Δα - t - Δα	Δα _t + Δα - t - Δα	- .164	Δα _t + Δα - t - Δα	Δα _t + Δα - t - Δα	- .082	Δα _t + Δα - t - Δα	Δα _t + Δα - t - Δα	- .082	Δα _t + Δα - t - Δα	Δα _t + Δα - t - Δα	- .082	Δα _t + Δα - t - Δα	Δα _t + Δα - t - Δα

TABLE XIV (Continued)

Computation of Chronometer Corrections: Sheet 3

Set 2 (Continued) $d\psi = 0.0183$ $d\epsilon = -0.0648$

Star: 1575 14 Pegasi			835 π Pegasi			844 β Lacertae			1590 38 Pegasi		
Level: W 34.1 E 79.0			W 72.0 E 27.1			W 35.0 E 80.1			W 71.5 E 26.4		
71.0 26.3			34.1 79.0			71.1 26.0			33.9 79.1		
+ 36.9 = 52.7			+ 37.9 = 51.9			+ 36.1 = 54.1			+ 37.6 = 52.7		
-15.8			-14.0			-18.0			-15.1		
b = 0.23621			= 0.20930			= 0.26910			= 0.22574		
B 1.13708			1.18331			1.58981			1.17358		
21 ^h 48 ^m			22 ^h 08 ^m			22 ^h 22 ^m			22 ^h 28 ^m		
47 48			07 09			21 22			27 28		
s	s	Sums	s	s	Sums	s	s	Sums	s	s	Sums
24.15	59.57	83.72	19.25	19.80	39.05	1.22	72.22	73.44	30.46	67.80	98.26
25.39	58.18	83.57	20.70	18.63	39.38	2.91	70.64	71.55	31.50	66.65	98.15
26.86	56.68	83.54	21.83	17.53	39.36	4.30	69.73	74.03	32.50	65.40	97.90
27.73	55.39	83.12	23.27	16.26	39.53	5.63	67.87	73.50	33.14	64.44	97.58
28.73	54.09	82.82	24.42	14.42	38.84	7.30	64.32	71.62	34.47	63.15	97.62
29.78	52.68	82.46	25.41	13.50	38.91	9.20	62.59	71.79	36.19	62.17	98.36
30.60	51.75	82.35	26.88	12.48	39.36	11.06	60.63	71.69	37.12	61.08	98.20
31.82	50.79	82.61	27.80	11.34	39.14	12.41	59.22	71.66	38.48	59.96	98.44
32.60	49.18	81.78	29.39	10.37	39.76	14.09	57.94	72.03	39.89	58.73	98.61
33.61	48.11	81.72	30.56	9.06	39.62	15.79	55.74	71.53	40.81	57.00	97.81
Mean		82.769	Mean		39.295	Mean		72.284	Mean		98.093
Mean/2		41.385	Mean/2		19.648	Mean/2		36.142	Mean/2		49.047
		11.385			19.648			06.142			19.047
		+ 0.067			+ 0.070			+ 0.095			+ 0.069
		= 0.019			= 0.019			= 0.026			= 0.019
		= 0.269			= 0.248			= 0.428			= 0.265
		11.164			19.451			05.783			18.832
		09.863			18.047			04.881			17.470
		+ 0.003			+ 0.003			+ 0.006			+ 0.004
		= 0.909			= 0.908			= 0.908			= 0.907
		= .389			= .493			+ .012			= .451
		$\Delta\alpha$			$\Delta\alpha$			$\Delta\alpha$			$\Delta\alpha$
		$\Delta\alpha$			$\Delta\alpha$			$\Delta\alpha$			$\Delta\alpha$
		$\Delta\alpha$			$\Delta\alpha$			$\Delta\alpha$			$\Delta\alpha$
		$\Delta\alpha$			$\Delta\alpha$			$\Delta\alpha$			$\Delta\alpha$
		$\Delta\alpha$			$\Delta\alpha$			$\Delta\alpha$			$\Delta\alpha$
		$\Delta\alpha$			$\Delta\alpha$			$\Delta\alpha$			$\Delta\alpha$
		$\Delta\alpha$			$\Delta\alpha$			$\Delta\alpha$			$\Delta\alpha$
		$\Delta\alpha$			$\Delta\alpha$			$\Delta\alpha$			$\Delta\alpha$
		$\Delta\alpha$			$\Delta\alpha$			$\Delta\alpha$			$\Delta\alpha$
		$\Delta\alpha$			$\Delta\alpha$			$\Delta\alpha$			$\Delta\alpha$
		$\Delta\alpha$			$\Delta\alpha$			$\Delta\alpha$			$\Delta\alpha$
		$\Delta\alpha$			$\Delta\alpha$			$\Delta\alpha$			$\Delta\alpha$
		$\Delta\alpha$			$\Delta\alpha$			$\Delta\alpha$			$\Delta\alpha$
		$\Delta\alpha$			$\Delta\alpha$			$\Delta\alpha$			$\Delta\alpha$
		$\Delta\alpha$			$\Delta\alpha$			$\Delta\alpha$			$\Delta\alpha$
		$\Delta\alpha$			$\Delta\alpha$			$\Delta\alpha$			$\Delta\alpha$
		$\Delta\alpha$			$\Delta\alpha$			$\Delta\alpha$			$\Delta\alpha$
		$\Delta\alpha$			$\Delta\alpha$			$\Delta\alpha$			$\Delta\alpha$
		$\Delta\alpha$			$\Delta\alpha$			$\Delta\alpha$			$\Delta\alpha$
		$\Delta\alpha$			$\Delta\alpha$			$\Delta\alpha$			$\Delta\alpha$
		$\Delta\alpha$			$\Delta\alpha$			$\Delta\alpha$			$\Delta\alpha$
		$\Delta\alpha$			$\Delta\alpha$			$\Delta\alpha$			$\Delta\alpha$
		$\Delta\alpha$			$\Delta\alpha$			$\Delta\alpha$			$\Delta\alpha$
		$\Delta\alpha$			$\Delta\alpha$			$\Delta\alpha$			$\Delta\alpha$
		$\Delta\alpha$			$\Delta\alpha$			$\Delta\alpha$			$\Delta\alpha$
		$\Delta\alpha$			$\Delta\alpha$			$\Delta\alpha$			$\Delta\alpha$
		$\Delta\alpha$			$\Delta\alpha$			$\Delta\alpha$			$\Delta\alpha$
		$\Delta\alpha$			$\Delta\alpha$			$\Delta\alpha$			$\Delta\alpha$
		$\Delta\alpha$			$\Delta\alpha$			$\Delta\alpha$			$\Delta\alpha$
		$\Delta\alpha$			$\Delta\alpha$			$\Delta\alpha$			$\Delta\alpha$
		$\Delta\alpha$			$\Delta\alpha$			$\Delta\alpha$			$\Delta\alpha$
		$\Delta\alpha$			$\Delta\alpha$			$\Delta\alpha$			$\Delta\alpha$
		$\Delta\$									

TABLE XIV (Continued)

Computation of Chronometer Corrections: Sheet 4

Set 3 : 7 August 1961 $d\psi = 0.0806$ $d\epsilon = 0.0641$

Star: 1600 BD ⁺ Lacertae			869 \odot Andromedae			1610 12 Andromedae			1616 15 Andromedae		
Level: W 68.5 E 22.9			W 33.9 E 79.8			W 70.8 E 25.0			W 33.0 E 78.5		
+ 30.1 75.9			68.2 22.3			32.0 77.5			69.6 23.9		
+ 38.4 = 53.0			+ 34.3 = 57.5			+ 38.8 = 52.5			+ 36.6 = 54.6		
= 14.6			= 23.2			= 13.7			= 18.0		
= 0.21827			= 0.34684			= 0.20482			= 0.26910		
B 1.24816			1.34722			1.26773			1.30585		
22h 53m			23h 00m			23h 19m			23h 32m		
52 54			59 02			18 19			31 33		
S	S	Sums	S	S	Sums	S	S	Sums	S	S	Sums
18.72	16.01	34.73	22.13	61.00	83.13	15.82	52.95	68.77	57.68	37.74	95.42
20.09	14.20	34.29	23.42	59.70	83.12	17.06	51.66	68.72	58.57	36.41	94.98
21.09	13.30	34.39	24.38	58.42	82.80	18.08	50.40	68.48	59.78	34.91	94.69
22.20	11.96	34.16	25.64	56.81	82.45	19.25	49.07	68.32	61.26	33.77	95.03
23.47	10.68	34.15	27.01	55.50	82.51	20.68	47.64	68.32	62.06	32.50	94.56
25.75	9.50	35.25	28.27	54.60	82.87	22.30	46.51	68.81	63.58	31.05	94.63
26.87	8.52	35.19	30.02	53.40	83.43	23.12	45.21	68.33	65.85	29.59	95.44
28.67	7.22	35.89	31.45	52.05	83.49	24.56	43.85	68.43	67.20	28.86	96.06
30.21	5.92	36.13	32.49	50.62	83.12	25.77	42.72	68.49	67.92	27.51	95.43
31.38	4.93	36.31	33.47	49.40	82.87	26.65	41.46	68.11	68.80	26.05	94.85
Mean		35.049	Mean		82.979	Mean		68.478	Mean		95.109
Mean/2		17.525	Mean/2		41.490	Mean/2		34.239	Mean/2		47.554
l		17.525			11.490			04.239			47.554
k		+ 0.073			+ 0.079			+ 0.074			+ 0.076
Bb		= 0.020			= 0.022			= 0.020			= 0.021
t		= 0.272			= 0.467			= 0.260			= 0.351
22h 53m		17.306	23h 00m		11.080	23h 19m		04.033	23h 32m		47.258
22h 53m		16.548	23h 00m		10.431	23h 19m		02.947	23h 32m		45.723
$\Delta\alpha$		+ .008			+ .008			+ .008			+ .008
Δt		= 0.907			= 0.907			= 0.907			= 0.907
$\Delta t - \Delta\alpha - \Delta t$		+ .157			+ .266			= .168			= .621

TABLE XIV (Continued)

Computation of Chronometer Corrections: Sheet 5

Set 3 (Continued)										Set 4 : 7 August 1961									
Star: 1619 γ Andromedae					1629 ψ Pegasi					4 22 Andromedae					1005 δ Andromedae				
Level: W 69.6 E 23.9					W 31.5 E 77.0					W 69.7 E 23.0					W 32.0 E 78.0				
32.0 77.8					69.4 23.9					32.2 78.0					69.2 23.6				
+ 37.6 - 53.9					+ 37.9 - 53.1					+ 37.5 - 55.0					+ 37.2 - 54.4				
- 16.3					- 15.2					- 17.5					- 17.2				
- 0.24368					- 0.22724					- 0.26162					- 0.25714				
1.38935					1.06478					1.42835					1.24289				
23 ^h 38 ^m					23 ^h 55 ^m					00 ^h 08 ^m					00 ^h 16 ^m				
37 39					54 56					07 09					15 16				
s	s	s	Sums	s	s	s	Sums	s	s	s	Sums	s	s	s	Sums				
39.49	26.51	66.00	54.74	45.90	100.64	22.38	20.90	31.34	70.58	101.92									
40.55	25.00	65.55	55.92	44.80	100.72	23.97	19.14	32.44	69.50	101.94									
42.69	23.66	66.35	57.22	43.79	101.01	25.00	17.70	33.64	68.37	102.01									
43.79	22.05	65.84	58.13	42.22	100.35	26.15	16.60	35.12	67.04	102.16									
45.51	20.89	66.40	59.01	41.21	100.22	27.21	15.15	36.37	65.76	102.13									
46.73	19.60	66.33	60.64	40.20	100.84	28.67	13.75	37.63	64.48	102.11									
47.82	17.80	65.62	62.00	39.02	101.02	30.58	11.70	39.06	63.00	102.06									
49.68	16.60	66.28	63.71	37.61	101.32	32.36	10.43	40.33	61.94	102.27									
50.89	15.06	65.94	64.65	36.82	101.47	33.82	9.28	41.81	61.23	103.04									
52.11	13.69	65.80	65.62	35.72	101.34	34.76	7.73	42.99	59.93	102.92									
Mean		66.011	Mean		100.893	Mean		Mean		102.256									
		33.006			50.447					51.128									
Mean/2		33.006	Mean/2		50.447	Mean/2		Mean/2		21.128									
1		+ 0.081			+ 0.064	+ 0.084		+ 0.073		+ 0.073									
k		- 0.022			- 0.018	- 0.023		- 0.020		- 0.020									
Bb		- 0.339			- 0.242	- 0.374		- 0.320		- 0.320									
t	23 ^h 38 ^m	32.726	23 ^h 55 ^m	55 ^m	50.251	00 ^h 08 ^m	08 ^m	00 ^h 16 ^m	16 ^m	20.861									
α	23 ^h 38 ^m	31.962	23 ^h 55 ^m	55 ^m	48.601	00 ^h 08 ^m	08 ^m	00 ^h 16 ^m	16 ^m	19.763									
$\Delta\alpha$		+ .009			+ .007	+ .010				+ .008									
Δt		- 0.906			- 0.906	- 0.906				- 0.906									
$\alpha + \Delta\alpha - t - \Delta t$		+ .151			- .737	+ .082				- .184									

TABLE XIV (Continued)

Computation of Chronometer Corrections: Sheet 7

Set 5 : 8 August 1961 $d\psi = 0.1254$ $d\epsilon = -0.0537$

Star: 1521 γ Cygni			1535 42 Cygni			792 β Cygni			1558 σ Cygni		
Level: W 68.2 E 27.0			W 69.5 E 26.2			W 68.5 E 25.0			W 41.0 E 84.1		
+ 28.5 - 55.0			+ 29.7 - 56.6			+ 27.5 - 59.0			+ 27.9 - 58.7		
- 26.5			- 26.9			- 31.5			- 30.9		
- 0.39618			- 0.40216			- 0.47092			- 0.46196		
1.21585			1.23870			1.38192			1.29094		
19h 54m			20h 27m			21h 03m			21h 15m		
53 55			27 28			02 04			15 16		
s	s	Sums	s	s	Sums	s	s	Sums	s	s	Sums
54.60	56.69	111.29	3.40	48.52	51.92	38.34	32.17	70.51	4.88	52.13	57.01
55.16	55.58	110.74	4.74	48.09	52.83	39.60	31.23	70.83	5.62	50.15	55.77
56.78	54.33	111.11	6.14	45.80	51.94	41.91	30.00	71.91	7.34	48.93	56.27
58.01	52.94	110.95	7.40	44.60	52.00	43.12	28.31	71.43	8.69	47.62	56.31
59.01	51.47	110.48	8.48	43.26	51.74	44.12	26.79	70.91	10.03	46.53	56.56
60.94	50.48	111.42	10.10	42.00	52.10	45.71	25.63	71.34	11.34	45.00	56.34
61.83	49.20	111.03	11.23	40.80	52.03	47.09	24.15	71.24	12.60	43.68	56.28
63.37	48.30	111.67	12.59	39.42	52.01	48.70	23.00	71.70	13.96	42.69	56.15
64.39	47.42	111.81	13.99	38.29	52.28	49.79	21.54	71.33	15.37	40.97	56.34
66.02	45.56	111.58	15.42	36.74	52.16	51.08	20.03	71.11	16.78	39.62	56.40
Mean		111.208	Mean		52.101	Mean		71.231	Mean		56.393
Mean/2		55.604	Mean/2		26.050	Mean/2		35.616	Mean/2		28.196
1		55.604			56.050			35.616			58.196
k		+ 0.071			+ 0.072			+ 0.081			+ 0.075
Bb		- 0.020			- 0.020			- 0.022			- 0.021
t		- 0.482			- 0.498			- 0.651			- 0.596
α	19h 54m	55.173	20h 27m		55.604	21h 03m		35.024	21h 15m		57.654
$\Delta\alpha$	19h 54m	53.184	20h 27m		53.883	21h 03m		33.730	21h 15m		55.920
Δt		+ .007			+ .007			+ .008			+ .008
$\alpha + \Delta\alpha - t - \Delta t$		- 2.151			- 2.153			- 2.155			- 2.156
		+ .169			+ .439			+ .869			+ .430

TABLE XIV (Continued)

Computation of Chronometer Corrections: Sheet 8

Set 5 : (Continued)

Set 6 : 8 August 1961

Star: 1575 14 Pegasi			844 β Lacertae			835 π Pegasi			1590 38 Pegasi		
Level: W 40.5 E 84.2			W 40.5 E 84.5			W 68.9 E 25.0			W 71.0 E 25.0		
68.8 25.0			69.0 25.0			40.0 83.8			40.0 83.1		
+ 28.3 - 59.2			+ 28.5 - 59.5			+ 28.9 - 58.8			+ 31.0 - 58.1		
- 30.9			- 31.0			- 29.9			- 27.1		
b - 0.46196			- 0.46345			- 0.44700			- 0.40514		
B 1.13708			1.58981			1.18331			1.17358		
21h 48m			22h 22m			22h 08m			22h 28m		
47 48			20 22			07 08			27 29		
S	s	Sums	S	s	Sums	S	s	Sums	S	s	Sums
25.39	58.69	84.08	59.75	74.23	133.98	29.90	70.61	100.51	31.22	68.46	00.68
27.11	57.88	84.99	61.22	72.24	133.46	31.67	69.87	101.54	32.82	67.13	99.96
28.20	56.40	84.60	63.06	70.62	133.68	32.07	68.39	100.46	33.75	65.62	99.37
29.30	55.37	84.67	64.64	69.10	133.74	33.38	67.56	100.94	34.66	64.70	99.36
30.64	54.67	85.31	65.92	67.57	133.49	34.77	65.70	100.47	35.68	63.54	99.22
31.52	53.04	84.56	67.89	65.82	133.71	35.95	64.24	100.19	37.40	62.40	99.80
32.80	51.90	84.70	69.17	64.12	133.29	37.17	63.53	100.70	38.43	61.60	100.03
34.27	50.78	85.10	70.80	61.94	132.74	38.41	61.79	100.20	35.49	59.96	99.45
35.30	49.80	85.10	72.86	60.27	133.13	39.54	60.48	100.02	41.00	58.88	99.28
36.14	48.12	84.26	74.50	59.34	133.84	41.09	59.40	100.49	42.10	58.41	100.51
Mean		84.737	Mean		133.506	Mean		100.552	Mean		99.726
Mean/2		42.368	Mean/2		66.753	Mean/2		50.276	Mean/2		49.863
		12.368			06.753			20.276			19.863
		+ 0.067			+ 0.095			+ 0.070			+ 0.069
		- 0.019			- 0.026			- 0.019			- 0.019
		- 0.525			- 0.737			- 0.529			- 0.475
Bb 21h 48m		11.891	22h 08m		06.085	22h 08m		19.798	22h 08m		19.438
21h 48m		09.883	22h 22m		04.910	22h 08m		18.072	22h 28m		17.500
$\Delta\alpha$		+ .008			+ .010			+ .009			+ .009
Δt		- 2.158			- 2.160			- 2.160			- 2.161
$\alpha + \Delta\alpha - t - \Delta t$		+ .158			+ .995			+ .443			+ .232

TABLE XIV (Continued)

Computation of Chronometer Corrections: Sheet 9

Set 6 : (Continued)

 $d\psi = 0.1280$ $de = -0.0524$

852 10 Lacertae			853 13 Lacertae			869 O Andromedae			1610 12 Andromedae		
Level: W 40.2 E 83.4			W 70.8 E 25.0			W 41.0 E 85.0			W 68.8 E 24.7		
69.6 25.2			40.0 84.0			68.8 24.8			40.0 84.2		
+ 29.4 - 58.2			+ 30.8 - 59.0			+ 27.8 - 60.2			+ 28.8 - 59.5		
- 28.8			- 28.2			- 32.4			- 30.7		
- 0.43056			- 0.42159			- 0.48438			- 0.45896		
B 1.28378			1.33707			1.34722			1.26773		
22 ^h 37 ^m			22 ^h 42 ^m			23 ^h 00 ^m			23 ^h 19 ^m		
36 38			41 43			59 61			18 19		
s	s	Sums	s	s	Sums	s	s	Sums	s	s	Sums
42.41	28.71	71.17	29.91	21.83	51.74	18.01	67.36	85.37	12.71	57.10	69.81
43.85	27.51	71.36	31.39	21.22	52.61	19.14	65.18	84.32	14.21	56.07	70.28
44.78	26.29	71.07	32.29	20.20	52.49	20.78	64.09	84.87	15.58	54.80	70.38
45.90	25.30	71.20	33.08	18.52	51.60	21.98	62.80	84.78	16.58	53.62	70.20
47.83	23.80	71.63	34.86	16.86	51.72	23.68	61.01	84.69	17.76	52.00	69.76
49.67	22.05	71.72	36.43	15.62	52.05	25.12	59.81	84.93	19.30	51.08	70.38
50.46	20.93	71.39	37.83	14.00	51.83	26.70	58.74	85.44	20.58	49.50	70.08
51.69	19.90	71.39	39.24	12.84	52.08	27.93	57.11	85.04	21.83	48.24	70.07
53.18	18.80	71.98	40.55	11.58	52.13	29.26	55.79	85.10	23.48	47.28	70.76
54.39	17.40	71.79	42.33	10.18	52.51	30.50	54.52	85.02	24.85	45.78	70.53
Mean		71.485	Mean		52.076	Mean		84.956	Mean		70.225
Mean/2		35.743	Mean/2		26.038	Mean/2		42.478	Mean/2		35.113
		35.743			26.038			12.478			05.113
l		+ 0.075			+ 0.078			+ 0.079			+ 0.074
k		- 0.021			- 0.022			- 0.022			- 0.020
Bb		- 0.553			- 0.564			- 0.653			- 0.582
t	22 ^h 37 ^m	35.244	22 ^h 42 ^m		25.530	23 ^h 00 ^m		11.882	23 ^h 19 ^m		04.585
d	22 ^h 37 ^m	33.506	22 ^h 42 ^m		23.998	23 ^h 00 ^m		10.448	23 ^h 19 ^m		02.967
Δd		+ .010			+ .010			+ .010			+ .010
Δ^t		- 2.161			- 2.161			- 2.163			- 2.164
$\Delta^t + \Delta d - t - \Delta t$		+ .433			+ .640			+ .739			+ .556

TABLE XV

Longitude Computation: Set 1

Star	A	$\alpha + \Delta\alpha$ $-t - \Delta t$	AA	$A(\alpha + \Delta\alpha$ $-t - \Delta t)$	Aa	$\Delta \lambda$	V
733	-.3255	+.332	.1060	-.1081	-.056	-.388	+.448
738	-.2745	-.392	.0754	-.1076	-.047	+.345	-.285
1521	+.1067	+.467	.0114	+.0498	+.018	-.449	+.509
757	-.1680	-.297	.0282	+.0499	-.029	+.268	-.208
1535	+.0795	-.422	.0063	-.0335	+.014	+.436	-.376
792	-.0912	-.164	.0083	+.0150	-.016	+.148	-.088
Sums	-.6730	-.476	.2356	+.0807		+.360	.000

Normal Equations:

$$\begin{aligned} 6\Delta\lambda a + .6730a - .476 &= 0 \\ .2356a - .0807 &= 0 \end{aligned}$$

Solution:

$\Delta\lambda a$	a	m
<u>6.0000</u>	+.6730	-.476
	<u>.16011</u>	-.02731

Back Solution:

-.11217	
+.07933	+.17057
+.06020	+.17057

$$\begin{aligned} a &= +.17057 \\ \Delta\lambda a &= +.06020 \end{aligned}$$

[VV]= .73339

$$\begin{aligned} m_s &= \pm .6745 \left(\frac{.73339}{4} \right)^{\frac{1}{2}} = \pm .6745 (.18335)^{\frac{1}{2}} = \pm .6745 (.42819) \\ &= \pm .28881 \end{aligned}$$

TABLE XV (Continued)

Longitude Computation: Set 2

Star	A	$\alpha + \Delta\alpha$ $-t - \Delta t$	AA	$A(\alpha + \Delta\alpha)$ $-t - \Delta t$	Aa	$\Delta\lambda$	v
1558	+.0172	-.365	.0003	-.0063	-.016	+.349	-.058
807	-.1608	-.082	.0259	+.0132	+.150	+.232	+.059
1575	+.2006	-.389	.0402	-.0780	-.187	+.202	+.089
835	+.1455	-.493	.0212	-.0717	-.136	+.357	-.066
844	-.3389	+.012	.1149	-.0041	+.316	+.304	-.013
1540	+.1571	-.451	.0247	-.0709	-.146	+.305	-.014
Sums	+.0207	-1.768	.2272	-.2178		+1.749	-.003

Normal Equations:

$6 \Delta\lambda a - .0207a - 1.768 = 0$
 $.2272a + .2178 = 0$

Solution:

$\Delta\lambda a$	a	n
<u>6.0000</u>	-.0207	-1.768
	<u>.22713</u>	+ .21170

Back Solution:

+.00345	
+.29467	-.93207
+.29145	-.93207

$a = -.93207$
 $\Delta\lambda a = +.29145$

$[VV] = .01949$

$m_s = \pm .6745 \left(\frac{(.01949)}{4} \right)^{\frac{1}{2}} = \pm .6745 (.00487)^{\frac{1}{2}} = \pm .6745 (.06979)$
 $= \pm .04073$

TABLE XV (Continued)

Longitude Computation: Set 3

Star	A	$\alpha + \Delta\alpha$ $-\lambda - \Delta\lambda$	AA	$A(\alpha + \Delta\alpha$ $-\lambda - \Delta\lambda)$	Aa	$\Delta\lambda$	n
1600	+.0682	+.157	.0047	+.0107	-.147	-.304	+.373
869	-.0498	+.266	.0025	-.0132	+.108	-.158	+.227
1610	+.0449	-.168	.0020	-.0075	-.097	+.071	-.002
1616	-.0005	-.621	.0000	+.0003	+.001	+.622	-.553
1619	-.1000	+.151	.0100	-.0151	+.216	+.065	+.004
1629	+.2868	-.737	.0823	-.2114	-.619	+.118	-.049
Sums	+.2496	-.952	.1015	-2362		+.414	,000

Normal Equations:

$$6\Delta\lambda a - (+.2496)a + (-.952) = 0$$

$$(+.1015)a - (-.2364) = 0$$

Solution:

$\Delta\lambda a$	a	n
<u>6.0000</u>	-.2496	-.952
	<u>.09112</u>	+.19680

Back Solution:

+.04160	
+.15867	-2.15978
+.06882	-2.15978

$$a = -2.15978$$

$$\Delta\lambda a = +.06882$$

$$[VV] = .498888$$

$$ms = \pm .6745 \left(\frac{[VV]}{n-2} \right)^{\frac{1}{2}} = \pm .6745 (.124722)^{\frac{1}{2}}$$

$$= \pm (.6745)(0.111679) = \pm .07533$$

TABLE XV (Continued)

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Longitude Computation: Set 3 (Rejecting Large Residuals)

Star	A	$\alpha + \Delta\alpha$ $-t - \Delta t$	AA	$A(\alpha + \Delta\alpha)$ $-t - \Delta t$	Aa	$\Delta\lambda$	✓
1600		REJECTED					
869		REJECTED					
1610	+ .0449	- .168	.0020	- .0075	- .105	+ .063	+ .008
1616		REJECTED					
1619	- .1000	+ .151	.0100	- .0151	+ .233	+ .082	- .011
1629	+ .2868	- .737	.0823	- .2114	- .668	+ .069	+ .002
Sums	+ .2317	- .754	.0943	- .2362		+ .214	- .001

Normal Equations:

$$3\Delta\lambda_a - (+.2317)_a + (-.754) = 0$$

$$(+.0943)_a - (-.2362) = 0$$

Solution:

$\Delta\lambda a$	a	n
<u>3.0000</u>	- .2317	- .754
	<u>.07641</u>	+ .17797

Back Solution:

+ .07723	
+ .25133	-2.32914
+ .07145	-2.32914

$$a = -2.32914$$

$$\Delta\lambda_a = +0.07145$$

$$[VV] = .0189$$

$$m_s = \pm .6745 \left(\frac{[VV]}{n-2} \right)^{\frac{1}{2}} = \pm .6745 (.0189)^{\frac{1}{2}}$$

$$= \pm (.6745)(.137477) = \pm .09273$$

TABLE XV (Continued)

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Longitude Computation: Set 4

Star	A	$\alpha + \Delta \alpha$ $-t - \Delta t$	AA	$A(\alpha + \Delta \alpha)$ $-t - \Delta t$	$A a$	$\Delta \lambda$	V
4	- .1465	+ .082	.0215	-.0120	+.295	+.213	-.058
1005	+ .0745	- .184	.0056	-.0137	-.150	+.034	+.121
17	- .3994	+ .658	.1595	-.2628	+.805	+.147	+.008
25	- .2101	+ .328	.0441	-.0689	+.424	+.096	+.058
33	+ .0380	- .356	.0014	-.0135	-.077	+.279	-.124
1030	- .4403	+ .727	.1939	-.3201	+.888	+.161	-.006
Sums	-1.0838	+1.255	.4260	-.6910		+.930	-.001

Normal Equations:

$$\begin{aligned} 6\Delta\lambda a &= (-1.0838)a + (+1.255) = 0 \\ (+.4260)a &- (-.6910) = 0 \end{aligned}$$

Solution:

$\Delta \lambda a$	a	n
<u>6.0000</u>	+1.0838	+1.255
	<u>.23023</u>	+ .46431

Back Solution:

-.18063	
-.20917	-2.01672
+.15511	-2.01672

$$\begin{aligned} a &= -2.01672 \\ \Delta\lambda a &= +.15511 \end{aligned}$$

$$[VV] = .03684$$

$$\begin{aligned} m_s &= \pm .6745 \left(\frac{[VV]}{m-2} \right)^{\frac{1}{2}} = \pm .6745 (.00921)^{\frac{1}{2}} \\ &= \pm (.6745)(.095969) = \pm .06473 \end{aligned}$$

TABLE XV (Continued)

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Longitude Computation: Set 5

Star	A	$\alpha + \alpha \Delta$ $-t - \Delta t$	AA	$A(\alpha + \Delta \alpha)$ $-t - \Delta t$	Aa	$\Delta \lambda$	\checkmark
1521	+ .1067	+ .169	.0114	+ .0180	- .183	- .352	- .151
1535	+ .0795	+ .439	.0063	+ .0349	- .136	- .575	+ .072
792	- .0912	+ .869	.0083	- .0793	+ .157	- .712	+ .209
1558	+ .0172	+ .430	.0003	+ .0074	- .030	- .460	- .043
1575	+ .2006	+ .158	.0402	+ .0317	- .344	- .502	- .001
844	- .3389	+ .995	.1148	- .3372	+ .582	- .413	- .090
Sums	- .0261	+ 3.060	.1813	- .3245		- 3.014	- .004

Normal Equations:

$$6\Delta\lambda a + .0261a + 3.060 = 0$$

$$.1813a + .3245 = 0$$

Solution:

$\Delta \lambda a$	a	n
<u>6.0000</u>	+ .0261	+ 3.060
	<u>.18119</u>	.31119

Back Solution:

- .00435	
- .51000	- 1.71748
- .50253	- 1.71748

$$a = -1.71748$$

$$\Delta\lambda a = - .50253$$

$$[VV] = .08162$$

$$m_s = \pm .6745 \left(\frac{.08162}{4} \right)^{\frac{1}{2}} = \pm .6745 (.02040)^{\frac{1}{2}} = \pm .6745 (.14283)$$

$$= \pm .09634$$

TABLE XV (Continued)

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Longitude Computation: Set 6

Star	A	$\alpha + \Delta\alpha$ $-t - \Delta t$	AA	$A(\alpha + \Delta\alpha)$ $-t - \Delta t$	Aa	$\Delta\lambda$	V
835	+.1455	+ .443	.0212	+.0645	-.295	- .738	+.134
1590	+.1571	+ .232	.0247	+.0364	-.318	- .550	-.054
852	+.0258	+ .433	.0007	+.0112	-.052	- .485	-.119
858	-.0377	+ .640	.0014	-.0241	+.076	- .564	-.040
869	-.0498	+ .739	.0025	-.0368	+.101	- .638	+.034
1610	+.0449	+ .556	.0020	+.0250	-.091	- .647	+.043
Sums	+.2858	+3.043	.0525	+.0762		-3.622	-.002

Normal Equations:

$$6\Delta\lambda a - .2858a + 3.043 = 0$$

$$.0525a - .0762 = 0$$

Solution:

$\Delta\lambda a$	a	n
<u>6.0000</u>	-.2858	+3.043
	<u>.03889</u>	+ .07875

Back Solution:

+.04768	
-.50717	-2.02494
-.60362	-2.02494

$$a = -2.02494$$

$$\Delta\lambda a = - .60362$$

$$[VV] = .03964$$

$$m_s = \pm .6745 \left(\frac{.03964}{4} \right)^{\frac{1}{2}} = \pm .6745 (.00991)^{\frac{1}{2}} = \pm .6745 (.09955)$$

$$= \pm .06715$$

TABLE XV (Continued)

Final Longitude Computation

Set	a s	$\Delta\lambda$ s	[V] s	m s	V s
1	REJECTED *				
2	-0.93207	+0.29145	-.003	$\pm .041$	-.40908
3	-2.32914	+0.07145	-.001	$\pm .093$	-.18908
4	-2.01672	+0.15511	-.001	$\pm .065$	-.27274
5	-1.71748	-0.50253	-.004	$\pm .096$	+.38490
6	-2.02494	-0.60362	-.002	$\pm .067$	+.48599
Sum		-0.58814			-.00001
Mean		-0.11763			

$[VV] = .66182$
 $m = \pm \left(\frac{[VV]}{n} \right)^{\frac{1}{2}} = \left(\frac{.66182}{5} \right)^{\frac{1}{2}} = (.13236)^{\frac{1}{2}}$
 $= \pm 0.^s.363813$

Mean	$\Delta\lambda$	a	-00. ^s .118	
Assumed	λ	05 ^h 32 ^m	10. ^s .000	
Mean Observed	λ	05 ^h 32 ^m	09. ^s .882	$\pm 0.^s.364$
Converted to Arc		83° 02'	28".230	$\pm 5".460$

* Set 1 rejected since only 1 star within limits

TABLE XVI

RADIO TIME SIGNALS

Station, OSU Farms Date: 6-7 August 1961 Freq. of sig. WWV, 5 mc.

Sending time of signal 22h.	Local Chron. time of signal 18h	Corr'n. to re- duce to mean epoch	Local Chron. time 18h 55m	Sending time of signal 00h	Local Chron. time of signal 20h	Corr'n. to re- duce to mean epoch	Local Chron. time 20h 40m
25 ^m 30 ^s	55 ^m 00 ^s .80	+ .068	25.868	10 ^m 37 ^s	40 ^m 25.12	+ .052	44 ^s .172
31	01.81	+ .066	.876	38	26.12	+ .049	.169
32	02.81	+ .063	.873	39	.12	+ .047	.167
33	03.81	+ .060	.870	40	.12	+ .044	.164
34	04.82	+ .057	.877	41	.10	+ .041	.141
36	.82	+ .052	.872	42	.11	+ .038	.148
37	.81	+ .049	.859	43	.10	+ .036	.146
38	.85	+ .047	.897	47	.12	+ .025	.145
39	.86	+ .044	.904	48	.12	+ .022	.142
40	.87	+ .041	.911	49	.12	+ .019	.139
46	.86	+ .025	.885	50	.11	+ .016	.126
47	.87	+ .022	.892	52	.12	+ .011	.131
48	.88	+ .019	.899	53	.12	+ .008	.128
49	.87	+ .016	.886	54	.13	+ .005	.138
50	.87	+ .014	.884	55	.15	+ .003	.153
25 55	55 25.88	+ .000	.880	56	40 44.17	+ .000	.170
56	.89	- .003	.887	57	.18	- .003	.177
57	.88	- .005	.875	58	.16	- .005	.155
58	.88	- .008	.872	59	.17	- .008	.162
26 01	.90	- .016	.884	01	.18	- .014	.166
02	.90	- .019	.881	02	.18	- .016	.164
03	.90	- .022	.878	03	.17	- .019	.151
04	.90	- .025	.875	04	.18	- .022	.158
05	.91	- .027	.883	05	.18	- .025	.150
06	.90	- .030	.870	07	.18	- .030	.150
08	.90	- .036	.864	08	.18	- .033	.147
09	.91	- .038	.872	09	.19	- .036	.154
10	.90	- .041	.859	11	.19	- .041	.149
11	.90	- .044	.856	12	.18	- .044	.136
12	.90	- .047	.853	13	.19	- .047	.143
Mean: 25 ^m 55 ^s				10 ^m 56 ^s 0			
55 ^m 25 ^s .878				40 ^m 44 ^s .151			

TABLE XVI (Continued)

Date, 7 August 1961

Sending time of signal	Local Chron. time of signal	Corr'n. to re- duce to mean epoch	Local Chron. time	Sending time of signal	Local Chron. time of signal	Corr'n. to re- duce to mean epoch	Local Chron. time
02h	22 h	epoch	22h 35m	04h	01 h	epoch	01h 11m
05 35	35 41.99	+ .044	42.034	40 30	11 02.42	+ .049	20.469
36	42.99	+ .041	1.031	31	42	+ .047	.467
37	43.98	+ .038	1.018	32	41	+ .044	.454
38	44.99	+ .036	1.026	33	41	+ .041	.451
39	45.99	+ .033	1.023	34	44	+ .038	.478
40	47.00	+ .030	1.030	35	45	+ .036	.486
41	48.00	+ .027	1.027	36	42	+ .033	.455
42	49.00	+ .025	1.025	37	43	+ .030	.460
43	49.99	+ .022	1.012	38	43	+ .027	.457
44	51.00	+ .019	1.019	39	43	+ .025	.455
45	52.00	+ .016	1.016	40	45	+ .022	.472
47	54.00	+ .011	1.011	44	45	+ .011	.461
48	55.00	+ .008	1.008	45	46	+ .008	.468
49	56.01	+ .005	1.005	46	47	+ .005	.475
50	57.01	+ .003	1.003	47	48	+ .003	.483
05 51	35 58.01	+ .000	.000	48	11 20.47	+ .000	.470
52	.02	- .003	.017	49	48	- .003	.477
53	02	- .005	.015	52	48	- .011	.469
54	02	- .008	.012	53	49	- .014	.476
01	06	- .027	.033	54	49	- .016	.474
02	07	- .030	.040	01	50	- .036	.464
03	06	- .033	.027	02	50	- .038	.462
05	06	- .038	.022	03	50	- .041	.459
06	06	- .041	.019	04	50	- .044	.456
07	08	- .044	.036	05	50	- .047	.453
08	08	- .047	.033	06	50	- .049	.451
09	09	- .049	.041	07	50	- .052	.448
10	08	- .052	.028	08	50	- .055	.445
11	08	- .055	.025	09	51	- .057	.453
13	08	- .060	.020	10	51	- .060	.450
Mean: 05 ^m 51 ^s 0				35 ^m 58 ^s 022			
				40 ^m 48 ^s 0			
				11 ^m 20 ^s 463			

TABLE XVI (Continued)

Date, 8 August 1961

Sending time of signal	Local Chron time of signal	Corr'n to re- duce to mean epoch	Local Chron time	Sending time of signal	Local Chron time of signal	Corr'n to re- duce to mean epoch	Local Chron. time
23h	19h	epoch	19h38M	02h	23h	epoch	23h 04m
m s	m s	s	s	m s	m s	s	s
00 14	37 44.88	+ .060	06.940	26 2	04 05.70	+ .041	20.741
15	.88	.057	.937	3	.70	.038	.738
16	.89	.055	.946	4	.70	.036	.736
19	.90	.047	.947	5	.70	.033	.733
20	.90	.044	.944	6	.71	.030	.740
21	.90	.041	.941	7	.71	.027	.737
23	.91	.036	.946	8	.71	.025	.735
25	.92	.030	.952	9	.71	.025	.732
26	.91	.027	.937	10	.72	.019	.739
27	.91	.025	.935	11	.72	.016	.736
28	.91	.022	.932	12	.72	.014	.734
29	.91	.019	.929	13	.72	.011	.731
32	.92	.011	.931	14	.72	.008	.728
34	.92	.005	.925	15	.72	.005	.725
35	.92	.003	.923	16	.72	.003	.723
00 36	38 06.93	.000	.930	26 17	04 20.73	.000	.730
37	.93	- .003	.927	18	.74	-.003	.733
38	.95	- .005	.945	19	.74	-.005	.735
40	.96	- .011	.949	20	.75	-.008	.742
41	.96	- .014	.946	21	.76	-.011	.749
42	.95	- .016	.934	22	.76	-.014	.746
43	.96	- .019	.941	23	.76	-.016	.744
45	.97	- .025	.945	24	.76	-.019	.741
46	.96	- .027	.933	25	.77	-.022	.748
47	.95	- .030	.920	26	.77	-.025	.745
49	.99	- .036	.945	27	.78	-.027	.753
50	.98	- .038	.952	28	.78	-.030	.750
52	.99	- .044	.946	29	.78	-.033	.747
53	.99	- .047	.953	30	.79	-.036	.754
54	.99	- .049	.941	31	.79	-.038	.752
Mean:							
00 ^m 36 ^s 0				26 ^m 17 ^s 0			
38 ^m 06 ^s .939				04 ^m 20 ^s .739			

TABLE XVII

COMPARISON OF CHRONOMETER AND RADIO SIGNALS

Station: OSU Farms, Ohio Latitude: 40° 00' 00" 0 N Longitude: 5^h 32^m 10^s.000
 Chronometer No.: 2 E - 12304

Local date	6 August '61	7 August '61	7 August '61	7 August '61	8 August '61	8 August '61
75th Mer. St. time of signal	h m s 22 25 55.000	h m s 00 10 56.000	h m s 02 05 51.000	h m s 04 40 48.000	h m s 23 00 36.000	h m s 02 26 17.000
Chron. time of signal	18 55 25.878	20 40 44.151	22 35 58.022	01 11 20.463	19 38 06.939	23 04 20.739
Transmitting station	WWV	WWV	WWV	WWV	WWV	WWV
Frequency of signal	5 mc	5 mc	5 mc	5 mc	5 mc	5 mc
G. C. T. of signal	7 August	7 August	7 August	7 August	9 August	9 August
Date	h m s	h m s	h m s	h m s	h m s	h m s
Time	03 25 55.000	05 10 56.000	07 05 51.000	09 40 48.000	04 00 36.000	07 26 17.000
Sid. time of 0 ^h G.C.T.	21 01 06.149	21 01 06.149	21 01 06.149	21 01 06.149	21 08 59.265	21 08 59.265
Change in nutation	+ 00.000	+ 00.000	- 00.001	- 00.001	- 00.001	- 00.002
Cor. mean solar to Sid. time	+ 33.827	+ 51.078	+ 01 09.957	+ 01 35.410	+ 39.525	+ 01 13.313
G.S.T. of signal	24 27 34.987	26 12 53.238	28 08 07.115	30 43 29.558	25 10 14.789	28 36 29.576
Longitude of sta. (assumed)	5 32 10.000	5 32 10.000	5 32 10.000	5 32 10.000	5 32 10.000	5 32 10.000
Local Sidereal time	18 55 24.987	20 40 43.238	22 35 57.115	01 11 19.558	19 38 4.789	23 04 19.576
Chron. time of signal	18 55 25.878	20 40 44.151	22 35 58.022	01 11 20.463	19 38 6.939	23 04 21.739
Chron. correction	(fast)-.891	(fast)-.913	(fast)-.907	(fast)-.905	(fast)-2.150	(fast)-2.163
Rate per minute		+0.000 209	- 0.003 130	- 0.000 012		+ 0.000 063

TABLE XVIII
LONGITUDE STAR PROGRAM

	No.	Mag.	R.A. h m	δ ° ' "	R.D. ° ' "	Circle West ° ' "	Circle East ° ' "
S	1441	5.4	16 51.5	31 46.2	8 14.0	8 14.0	351 46.0
N	1448	6.3	17 03.9	43 52.2	-3 52.0	356 08.8	3 52.0
S	1456	5.4	17 19.2	32 31.3	7 28.9	7 28.9	352 31.1
N	1462	5.8	17 31.9	41 16.5	-1 16.3	358 43.7	1 16.3
S	672	4.8	17 54.9	37 15.6	2 44.6	2 44.6	357 15.4
S	1477	4.3	18 18.5	36 03.1	3 57.1	3 57.1	356 02.9
S	705	4.0	18 48.7	33 19.3	6 40.9	6 40.9	353 19.1
N	733	3.9	19 28.8	51 39.1	-11 38.9	348 21.1	11 38.9
N	738	4.6	19 35.4	50 08.1	-10 07.9	349 52.1	10 07.9
S	1521	4.0	19 54.9	34 59.0	5 01.2	5 01.2	354 58.8
N	757	4.0	20 12.4	46 37.6	-6 37.4	353 22.6	6 37.4
S	1535	5.9	20 27.9	36 19.7	3 40.5	3 40.5	356 19.5
N	788	4.0	20 55.8	41 01.2	-11 01.0	348 59.0	11 01.5
N	792	0.9	21 03.6	43 46.5	-3 46.3	356 13.7	3 46.3
S	1558	4.3	21 15.9	39 14.1	0 46.1	0 46.1	359 13.9
N	807	5.3	21 28.1	46 22.3	-6 22.1	353 37.9	6 22.1
N	811	5.1	21 35.4	40 14.5	-0 14.3	359 45.7	0 14.3
S	1575	5.0	21 48.2	29 59.7	10 00.5	10 00.5	349 59.5
S	835	4.4	22 08.3	32 59.4	7 00.8	7 00.8	352 59.2
N	844	4.6	22 22.1	52 02.1	12 01.9	347 58.1	12 01.9
S	1590	5.5	22 28.3	32 22.5	7 37.7	7 37.7	352 22.3
S	852	4.9	22 37.6	38 51.0	1 09.2	1 09.2	358 50.8
N	858	5.2	22 42	41 37.0	-1 36.8	358 23.2	1 36.8
S	1600	6.0	22 53.3	36 52.3	3 07.9	3 07.9	356 52.1
N	869	3.6	23 00.2	42 07.1	-2 06.9	357 53.1	2 06.9
S	1610	5.8	23 19.0	37 58.3	2 01.9	2 01.9	357 58.1
N	1616	5.5	23 32.8	40 01.4	-0 01.2	359 58.8	0 01.2
N	1619	4.3	23 38.5	44 07.1	-4 06.9	355 53.1	4 06.9
S	1629	4.8	23 55.8	24 55.6	15 04.6	15 04.6	344 55.4
N	4	5.1	0 08.3	45 51.4	-5 51.2	354 08.8	5 51.2
S	1005	4.5	0 16.3	36 34.2	3 26.0	3 26.0	356 34.0
N	17	3.7	0 34.8	53 40.9	-13 40.7	346 19.3	13 40.7
N	25	4.7	0 42.6	48 04.2	-8 04.0	351 56.0	8 04.0
S	33	3.9	0 54.6	38 17.4	+1 42.6	1 42.6	358 17.2
N	1030	5.3	1 05.7	54 43.7	-14 43.5	345 16.5	14 43.5
AZIMUTH STARS							
S	647	4.6	17 24.6	-5 03.0	45 03.2	45 03.2	314 56.8
N	670	4.9	17 42.7	72 10.3	-32 10.1	327 49.9	32 10.1
S	688	3.7	17 45.9	2 43.5	37 16.7	37 16.7	322 43.3
S	680	3.7	18 05.5	9 33.7	30 26.5	30 26.5	329 33.5
S	1484	5.4	18 34.6	9 05.7	30 54.5	30 54.5	329 05.5
S	725	5.1	19 16.0	11 31.6	28 28.6	28 28.6	331 31.4
S	778	4.5	20 31.6	14 56.1	25 04.1	25 04.1	334 55.9
N	1492	5.8	18 45.9	52 57.0	-12 56.8	347 03.2	12 56.8
N	675	5.0	17 51.2	76 58.5	-36 58.3	323 01.7	36 58.3

3. DETERMINATION OF GEODETIC LATITUDE AND LONGITUDE

3.1 - The Observations

The observations were made with the DKM-3 number 65303 (OSU 369215) by the method of all positions. The horizontal circle and optical micrometer were rotated eight times (45° and $38''$, respectively, for each rotation), and two observations were made on each face. Directions were sighted in one direction only, and the grid coordinates of the Astro Pillar, OSU Farms, were computed by the resection method outlined in Clark [5] p. 565. The observations were made under adverse conditions with temperatures ranging from -5°F to $+40^{\circ}\text{F}$, over a period of four days. An attempt was made to observe on several afternoons, but the lateral refraction was too great.

The points used for targets were "Medary Ventilator" and "West Stadium", as defined in [12]. Medary ventilator is located atop Medary School, 2500 Medary Street and is marked by a black and white striped vertical pole. West Stadium is marked by a lead plug countersunk into the concrete roof of the southwest tower of the OSU Football Stadium. The third target station used was "Union Antenna", which coordinates were obtained from reference [6] in the Ohio state coordinate system and reduced to the Columbus city coordinate system. Union Antenna is the WCBE-TV antenna atop the Leveque-Lincoln Tower at 50 West Broad Street in downtown Columbus. This antenna is supported by a heavy steel structure painted red and white

<u>DATUM</u>	<u>FROM</u>	<u>TO</u>	<u>DISTANCE (FT)</u>	<u>BEARING</u>
800 Feet	Asylum	Brewery	20,257.80	S 54° 12' 37".75 E
Correction			- 1.12	
State datum (by city system)			20,256.68	S 54° 12' 37".75 E
State datum (by state system)			<u>20,256.67</u>	<u>S 53° 53' 42".07 E</u>
Difference			0.01	0° 18' 55".68 CW

Corrections to State System (38 ppm to sea level + 17 ppm scale factor) = -1.12 feet

Using the preceding values, Mr. Watts computed the state coordinates of station "Bank" to be : $X = 1,859,715.73$, $Y = 715,165.53$. The line "Bank" back to "Asylum" was then computed in the state system and reduced to the 800 foot datum. Comparison with results of the original survey showed a misclosure of 0.06 feet and 0".13, which is within the accuracy limits of my observations.

The coordinates for station "Astro Pillar", OSU Farms were reduced to the state coordinate system as follows:

<u>DATUM</u>	<u>FROM</u>	<u>TO</u>	<u>DISTANCE (FT)</u>	<u>BEARING</u>
800 feet	Astro Pillar	Bank	18,884.18	S 37° 10' 04".53 E
(Sea level correction = 38 ppm + scale factor = 10 ppm)			- 0.90	- 0° 18' 55".68
			<u>18,883.28</u>	<u>S 36° 51' 08".85E</u>

The resulting coordinates are: $X = 1,848,390.36 \pm 0.25$
 $Y = 730,275.60 \pm 0.25$

3.3 Computation of Geodetic Coordinates

The computation in Table XX follows the format of refer-

ence [13] which includes all the tables necessary to complete the calculations. The final Geodetic Coordinates were determined to be:

Latitude = $40^{\circ} 00' 13''.076 \pm 0''.002$ (estimated)

Longitude = $83^{\circ} 02' 29''.287 \pm 0''.002$ (estimated)

These geodetic coordinates refer to the North American 1927 Datum, the Clark (1866) Ellipsoid, with origin at Meade's Ranch: Latitude $39^{\circ} 13' 26''.686$ North, Longitude $98^{\circ} 32' 30''.506$ West.

TABLE XIX

Computation of Grid Coordinates of Astro-Pillar

	A Union Antenna	S West Stadium	M Medary Ventilator
Y	99 907.553	113 993.689	118 932.302
ΔY	14 086.136		4 938.626
X	99 485.155	94 283.839	98 880.465
ΔX	- 5 201.316		-4596.626

ANGLE	o	'	"	sine
Bg. MS = $\arctan (1.074 \ 39957) =$	47	03	14.6	.7319 9681
Bg. AS = $\arctan (2.708 \ 18693) =$	69	43	59.9	.9380 9044
MSA = (Bg. MS + Bg. AS) =	116	47	14.5	-
c = $(360^\circ - \text{MSA})$	243	12	45.5	-
a = mean observed value =	31	10	30.5	.5176 5582
b = mean observed value =	43	46	10.6	.6917 6027
a + b + c =	318	09	26.6	-
k = $(u + V) = [360^\circ - (a+b+c)]$	41	50	33.4	.6670 8670
u = $\cotan^{-1} (2.016 \ 84576)$	26	22	23.9	.4442 1782
V = $(k - u) =$	15	28	09.5	.2667 2211
Bg. MP = $(\text{Bg. MS} - u) =$	20	40	50.7	.3531 6054
Bg. AP = $(\text{Bg. AS} - V) =$	54	15	50.4	.8117 1672
(a + u) =	57	32	54.4	.8438 4544
(b + V) =	59	14	20.1	.8593 0749
Bg. SP = $[\text{ASP} - (90^\circ + \text{Bg. SA})]$	10	29	39.8	.1821 3924

Sides

MS = 6,746.767 feet

AS = 15,015.755 feet

By law of sines:

SP = 5789.627 feet

MP = 10998.096 feet

AP = 18652.634
feet

FROM		Y	X
A	$\Delta Y = \text{AP} \sin (\text{Bg. AP}) = +15 \ 140.655$ $\Delta X = \text{AP} \cos (\text{Bg. AP}) = -10 \ 894.096$	115 048.208 -	- 88 591.059
M	$\Delta Y = \text{MP} \sin (\text{Bg. MP}) = -3 \ 884.094$ $\Delta X = \text{MP} \cos (\text{Bg. MP}) = -10 \ 289.409$	115 048.208 -	- 88 591.056
S	$\Delta Y = \text{SP} \sin (\text{Bg. SP}) = +1 \ 054.518$ $\Delta X = \text{SP} \cos (\text{Bg. SP}) = -5 \ 692.782$	115 048.207 -	- 88 591.057
Mean (City System):	Y = 115 048.208	X = 88 591.057	
(State System);	Y = 730 275.60	X = 1 848 390.36	

TABLE XX

Computation of Geodetic Coordinates

C = value of X for Central Meridian = 2,000,000.00 feet
R_b = constant for zone (Ohio south) = 26,027,071.12 feet
θ = mapping angle for the longitude of the station
R = radius for the latitude of the station = R_b * cos θ
ℓ = constant for zone (Ohio south) = .63451 95439
Δλ = θ + ℓ
λ = Central Meridian of zone - Δλ
Central Meridian of zone (Ohio south) = 82° 30' 00"000
TAN θ = (X - C) * (R_b - Y)
ΔR = computed R - tabulated R (nearest minute)
k = tabular difference for 1 second of latitude = 101.18850 feet
Δφ = ΔR * k

C	2 000 000.00	R _b	26 027 071.12
X	1 848 390.36	Y	730 275.60
X-C	- 151 609.64	R _b -Y	25 296 795.52
Tan θ	-.00599 32350	θ	- 1236"1786
θ	- 0° 20' 36"1786	Δλ	- 1948"212
COS θ	.99998 20409	Δλ	-0° 32' 28"212
R	25 297 249.83	C.M.	82° 30' 00"000
R (40° 00')	25 298 632.46		
ΔR	1 382.63		
Δφ	13"664		
φ	40° 00' 13"664	λ	83° 02' 28"212

Geodetic Coordinates:
Latitude: 40° 00' 13"664
Longitude: 83° 02' 28"212

TABLE XXI

Theodolite Observations: January 1962

Observer: Lt. Bennett

Recorder: Mrs. Bennett

Station: Astro-Pillar, OSU Farms

Instrument: DKM-3 #65303

Set Date Temp	Face	Medary Ventilator			Diff. 31° 10'	West Stadium			Diff. 43° 45'	Union Antenna			Residual	
		o	'	"		o	'	"		o	'	"	a	b
1 1/10 -5°F	R	000	23	53.5	24.3	31	34	17.3	65.1	75	20	22.9	6.2	-5.5
	L	180	23	58.0	30.0	211	34	28.0	64.5	255	20	32.5	0.5	-6.1
	L	180	24	03.0	32.0	211	34	35.0	55.4	255	20	30.4	-1.5	-15.2
	R	000	23	39.6	25.9	31	34	15.5	68.4	75	20	33.9	4.6	-2.2
2	R	044	56	44.4	38.3	76	07	22.7	62.7	119	53	35.4	7.8	-7.9
	L	224	56	47.0	50.5	256	07	37.5	65.0	299	53	42.5	20.0	-5.6
	L	224	56	51.6	37.2	256	07	28.8	70.1	299	53	38.9	7.5	-0.5
	R	044	57	01.4	30.6	76	07	32.0	66.3	119	53	38.3	0.1	-4.3
3 1/12 +15°F	R	090	06	58.3	33.3	121	17	31.5	63.4	165	03	34.9	2.7	-7.2
	L	270	07	06.0	31.1	301	17	37.1	67.7	345	03	44.8	0.6	-2.9
	L	090	07	05.9	27.4	301	17	33.3	67.2	345	03	40.5	-3.1	-3.4
	R	270	06	55.1	34.9	121	17	30.0	56.5	165	03	26.5	4.4	-13.9
4	R	134	59	48.6	33.9	166	10	22.5	69.8	209	56	32.3	3.4	-0.8
	L	314	59	54.5	33.9	346	10	28.4	72.9	29	56	41.3	3.4	2.3
	L	314	59	56.8	29.6	346	10	26.4	76.0	29	56	42.4	-0.9	5.4
	R	134	59	15.6	27.8	166	09	43.4	85.8	209	56	09.2	-2.7	15.2
5 1/13 +32°F	R	180	00	37.5	29.5	211	11	07.0	72.3	254	57	19.3	-1.0	1.7
	L	000	00	44.5	31.7	31	11	16.2	67.4	74	57	23.6	1.2	-2.8
	L	000	00	47.0	49.9	31	11	16.9	62.4	74	57	19.3	-0.6	-8.2
	R	180	00	40.1	21.5	211	11	01.6	71.2	254	57	12.8	-9.0	0.6
6	R	224	59	16.1	24.4	256	09	40.5	46.7	299	55	27.2	-6.1	-23.9
	L	044	58	20.0	27.5	76	08	47.5	73.7	119	55	01.2	-3.0	6.9
	L	044	58	18.5	30.4	76	08	48.9	66.6	119	54	55.5	-0.1	-4.0
	R	224	58	11.0	26.8	256	08	37.8	75.8	299	54	53.6	-4.3	5.2
7 1/14 +40°F	R	270	10	34.2	30.9	301	21	05.1	81.7	345	07	26.8	0.4	11.1
	L	090	10	42.9	33.2	121	21	16.1	82.3	165	07	38.4	2.7	11.7
	L	090	10	45.0	27.4	121	21	12.4	81.6	165	07	34.0	-3.1	11.0
	R	270	10	38.0	29.5	301	21	07.5	74.1	345	07	21.6	-1.0	3.5
8	R	314	59	25.6	29.4	346	09	55.0	81.6	29	56	16.6	-1.1	11.0
	L	134	59	31.1	29.7	166	10	00.8	81.8	209	56	22.6	-0.8	11.2
	L	134	59	32.5	32.0	166	10	04.5	81.2	209	56	25.7	1.5	10.6
	R	314	59	21.7	32.3	346	09	54.0	81.3	29	56	15.3	1.8	10.7

Sum

976.7

2258.5

Mean

30.5

70.6

$$a = 31^{\circ} 10' 30''.5$$

$$b = 43^{\circ} 46' 10''.6$$

$$a + b = 74^{\circ} 56' 41''.1$$

$$M_a = \pm 0''.96$$

$$M_b = \pm 1''.86$$

4. THE DEFLECTION OF THE VERTICAL

The component of the deflection of the vertical in the meridian (ξ) is obtained as follows:

$$\begin{aligned}\xi &= \text{Astro Latitude} - \text{Geodetic Latitude} \\ &= 40^{\circ} 00' 13''.401 - 40^{\circ} 00' 13''.076 \\ &= + 0''.225\end{aligned}$$

The component of the deflection of the vertical in the prime vertical (η) is obtained as follows:

$$\begin{aligned}\eta &= (\text{Astro Longitude} - \text{Geodetic Longitude}) \cos \phi \\ &= (83^{\circ} 02' 28''.230 - 83^{\circ} 02' 29''.287) \cos \phi \\ &= (- 01''.057) \cos \phi = 0''.806\end{aligned}$$

$$\text{Where: } \cos \phi = 0.76604$$

The accuracy of these deflection components is limited by the accuracy of the astronomic latitude and longitude:

$$\begin{aligned}\text{Latitude probable error} &= \pm 0''.146 ; \text{standard error} = \\ &\pm 0''.216\end{aligned}$$

$$\begin{aligned}\text{Longitude probable error} &= \pm 3''.683 ; \text{standard error} = \\ &\pm 5''.460\end{aligned}$$

The final values for the components of the deflection are thus found to be (within the limits defined by the accuracy of the astronomic coordinates):

$$\begin{aligned}\xi &= + 0'' 225 \pm 0'' 216 \\ \eta &= - 0'' 806 \pm 4'' 182\end{aligned}$$

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